A Simple Recipe for Monetarists from Deflationary Activists *

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I. Introduction

The failure of activist monetary policies in the late 1970s encouraged the advocates of the monetarists' policy rules. Namely monetarists' view that activist policies may lead the economy away from the optimal path prevailed. This view has conveyed into several policy rules such as the fixed money rule, the Freidman's k-percent rule, Taylor rule or, recently, inflation targeting. In the pioneering work of Barro-Gordon [1983], (hereafter BG), their model explicitly demonstrated the superiority of the "rule" to the "discretion" in terms of the social loss function. A popular understanding of the BG model is to support the strict monetary policy rules of the monetarist, and following studies mainly focused on *how* the rules must be formulated. Discretion is deemed as a silent part of the BG paper.

However, the model presented here reveals the further implication about activist monetary policies under the BG discretionary regime. In particular, our model illustrates the possibility of a deflation in a discretionary equilibrium even if the central bank prefers a positive rate of inflation. Under the dynamic formulation of the BG model, we have shown that the persistency of output gap with respect to the natural rate level may drag activist policies to zero or below zero-inflation rate. Thereafter, even under an activist policy, the policy maker may observe a deflationary price movement as she does under the rules; that is particularly so when she is remorselessly active.

This ambiguity in identifying policy stance arises in practical policy operations. First, a central bank under inflation targeting usually simultaneously adjusts monetary policy actively, to enhance the outcome. The adoption of the inflation target is overly emphasized and the other policy measures tend to be overlooked by the monetarists; the reality may be the result of the comprehensive activist policy package. As the results, even though activist policies may have a greater effect on the outcome, it is interpreted as a result of inflation targeting treated as a rule. Second, during the Greenspan's Fed chairmanship, the US economy was well controlled by the appropriate policy mix. Clarida et al [2000], and Judd and Rudebusch [1998] claimed that the stability was due to the interest rate rule with an inflation coefficient greater than unity. The monetarist interpretation of the successes is the triumph of the BG model's conclusion, such that Fed implicitly followed the Taylor rule. Contrary to this interpretation, high responsiveness of the nominal interest rate to the inflation rate can be viewed as the result from the active monetary policy or even by a remorseless activist's policy. Notably to say, the inclusion of the output gap in the explanatory part of the Taylor rule gives an enough room to the activist to "sneak into" the monetary policy rules. The fact that variants of Taylor-type rules fit U.S. data reasonably well means the stability of the feedback rules chosen by the central bank. When monetary policy acts as a stabilizer for the economy, however, no central bank has made a commitment to follow a Taylor rule. Rather, we think of this stability as the time-consistency of discretionary equilibrium. That is to say, the economy is in a discretion equilibrium, in which monetary policy becomes time-invariant and actual and privately expected feedback rules coincide. We define a discretionary policy regime as one in which the central bank chooses the cost-minimizing feedback rule given the public's expectations of feedback coefficients.

One of the main purposes of this paper is to demonstrate the possibility of prevalence of zero-inflation or deflation in the discretionary equilibria when the policy maker is remorselessly active. This critical aspect deceives the results commonly accepted in the context of the "rule versus discretion" literatures; namely, activist policy would result in the high-inflation, thereafter a remorseless activist would cause hyperinflationary economy.

In order to analyze the macroeconomic consequences of monetary policy regimes in a dynamic optimizing model, this paper employs a simple model in which monetary policy takes the form of inflation rate feedback rules instead of interest rate feedback rules.[1] This simple model yields the results that capture the above mentioned aspect. The contrast between the expectations formation process and the timing of moves assumed in this model and those in the previous literatures are essential for this purpose.

Our results show that there is a unique, efficient equilibrium under commitment and multiple equilibria under discretionary policy. The optimal monetary policy chosen under commitment exhibits a strict inflation targeting rule. Not surprisingly, consistent with the natural-rate hypothesis, any feedback rule is welfare-dominated by a strict inflation targeting rule whereby the inflation rate is kept at its target level (zero inflation in BG [1983]) at all dates. Then we investigate the properties of equilibrium in a discretionary regime.

Interestingly, two types of discretionary equilibria can be characterized as a deflationary equilibrium and a high-inflationary equilibrium. The central bank chooses active (passive) monetary policy in the former (in the latter) type of discretionary equilibrium, where we adopt these terminologies from Leeper [1991]. This same conclusion-- that there are multiple equilibria in discretionary regime is also obtained in the more complex model studied by King and Wolman [2004].[2]

We also analyzed a welfare comparison between two types of discretionary equilibria. The implied value of the loss function under discretion depends on the variance of the shocks and on the extent to which the central bank seeks to keep output above the natural rate. For most parameter values, the presence of large external shocks generates greater instability in the active equilibrium than in the passive equilibrium. In contrast, monetary policy generates higher inflation and greater instability in the passive equilibrium than in the active equilibrium when the central bank tries to target an unrealistically high level of output.

The rest of the paper is organized as follows. The next section lays out the basic model and explains the expectations formation process and the relative timing of moves assumed in this paper. After defining the concepts of rational expectations equilibrium and the optimum under commitment, Section 3 fully describes them in terms of the underlying parameters. In Section 4, it is shown that there exist two types of discretionary equilibria, a deflationary equilibrium and a high-inflationary equilibrium. In Section 5, the welfare consequences of alternative equilibria will be examined. Finally, Section 6 concludes the paper.

II. The Model

The model that we demonstrate in this paper is a standard neoclassical Phillips curve model in which monetary policy can have important real effects. The model's five key elements are: a loss function that the central bank should use to evaluate alternative paths for the economy, an aggregate supply relation, an expectations formation, a feedback rule for the rate of inflation, and an assumption on the timing of moves. The expectations formation of the public is based on the perceived decision rule of the central bank, as is the case in linear quadratic dynamic game. The timing of moves is so altered relative to the model of Dittmar, Gavin and Kydland [1999] (hereafter DGK), that the central bank takes into account the public's expectations formation (not the expected rate of inflation) when she chooses a feedback rule. In other respects, the model is similar to the models used in a number of recent analyses of inflation targeting, including Svensson [1997], [1999], DGK, and Dittmar and Gavin [2000].

The Central Bank's Intertemporal Loss Function

The first element of the model is the central bank's loss function. Following DGK, we specify the central bank's loss function as (1) where she minimizes expected value of weighted sum of output variability and inflation variability. Let y_τ denote output deviation from its natural rate (in logarithm) and π_τ denote the rate of inflation in period τ. For an optimal level of the output gap $\bar{y} > 0$, and the inflation rate π^*, and for arbitrary weight $0 \le \lambda \le 1$, the object of monetary policy is to minimize the expected value of a loss criterion of the form:

(1) $$L = \sum_{\tau=t}^{\infty} \beta^\tau E_t \left\{ \lambda (y_\tau - \bar{y})^2 + (1-\lambda)(\pi_\tau - \pi^*)^2 \right\}.$$

The central bank discounts future variability in the output gap and inflation by the factor $0 < \beta < 1$. The $\bar{y}$ in (1) does not indicate the natural rate of output, as discussed in BG. "*In the presence of unemployment compensation, income taxation, and the like, the natural unemployment rate will tend to exceed the efficient level---that is privately chosen quantities of marketable output and employment will tend to be too low*" (BG, p.591).

The Short-Run Phillips Curve

As the second element, we take advantage of DGK's Lucas supply function where the central bank is constrained by the aggregate supply relation of the form:

(2) $$y_\tau = \rho y_{\tau-1} + \alpha(\pi_\tau - \pi_\tau^e) + \eta_\tau,$$

where η_τ is an iid technology shock with mean zero and constant variance σ_η^2. The positive constants, ρ and α represent the degree of output persistence and the slope of the short-run Phillips curve. Current realizations of η_τ can be observed by neither the central bank nor the public. The inclusion of the logged value of output deviation is justified because of the wage contract, menu cost, etc., (DGK, p.24). Finally, π_τ^e denotes the public's expectation of π_τ in period τ, conditional on the information available in period τ.[3]

Expectations Formation of the Public

It is reasonable for the public to form an expectation about the central bank's *feedback rule.* More formally, the public expects inflation to be determined by the linear function:

(3) $$\pi_\tau = A_1^e + A_2^e y_{\tau-1},$$

where A_1^e, A_2^e are the *feedback coefficients* anticipated by the public. Here, we assume Markov property and characterize the public's perceived decision rule of the central bank as depending only on the value of the state variable, y_{t-1}, not the entire history.

Consider an economy where it is known that the central bank's policy choice is a function of readily observable macro variables. The public under such policy choice expects rather the function or strategy of the central bank than a particular value of the policy. For the case of Taylor rule, which sets the nominal interest rate as a linear function of a current inflation rate and a current measure of output relative to potential, the public expects the linear function chosen by the central bank, or equivalently, its inflation coefficient and output gap coefficient. In the context of the model just set out, the equation (3) expresses the public's perceived decision rule of the central bank as a function of the lagged output gap. The public's expectation of π_τ in period τ, conditional on the information available in period τ, π_τ^e can be written as

(4) $$\pi_\tau^e = A_1^e + A_2^e y_{\tau-1}.$$

The Central Bank's Reaction Function

We now obtain the Phillips curve (Lucas supply function) constraint consistent with the public's expectation. Substituting (3) into (2) gives:

(5) $$y_\tau = -\alpha A_1^e + (\rho - \alpha A_2^e) y_{\tau-1} + \alpha \pi_\tau + \eta_\tau .$$

Given the linear structural equation, (5), and the quadratic loss function of our model, (1), the central bank's reaction function (authority's strategy) takes the form of:

(6) $$\pi_\tau = A_1 + A_2 y_{\tau-1} .$$

Clearly, a set of *feedback coefficients* $[A_1, A_2]$ identifies the strategy space for the central bank. Without the Markov property of the public's perceived decision rule of the central bank, (3), the central bank's reaction function does not always take the form of (6). Note that the central bank's reaction function depends on the public's expectations formation as well as the loss function (1) and the constraint (2).

<u>The Timing of Moves</u>

Here, we will take advantage of a game-theoretical structure to discuss the interaction between the expectations formation of the public and the central bank's choice of feedback rule. The central bank's strategy space is the set of feedback rules of the form (6) (or, equivalently, that of *feedback coefficients* $[A_1, A_2]$) and its loss function is defined by (1). The strategy space of the public is the set of functions of the form (3) or, equivalently, the set of $[A_1^e, A_2^e]$, but its loss function has not to be specified explicitly.

The last element of the model is the timing of moves. The contrast between the timing of moves assumed here and that assumed in DGK is essential for the following argument. In this model, the central bank is assumed to choose its feedback rule *after* the public expects the inflation rate rule. In other words, the central bank takes into account the public's expectations formation when she chooses a feedback rule. In discretionary regime, the central bank chooses the optimal path for inflation subject to the constraint, (5). Before turning to a discretionary regime, it will be useful to consider a rational expectations equilibrium and the social optimum under commitment in the next section.

III. Rational Expectations Equilibrium and the Social Optimum under Commitment

Consider the economy which is described by the key elements, (1), (2), and (6). In the context of this economy, rational expectations hypothesis implies that the public forms inflation expectations rationally, in accordance with (6): $\pi_\tau^e = E_\tau \pi_\tau$. Therefore

(7) $\pi_\tau^e = A_1 + A_2 y_{\tau-1}$.

This condition implies that the process of expectations formation can be described by using the feedback rule chosen by the central bank itself. In contrast, this paper assumes expectations formation of the form (3). In our model, (7) holds in discretion equilibria, but $A_1 = A_1^e$ and $A_2 = A_2^e$ are equilibrium conditions and not assumptions.

In discretionary regime analyzed in the next section, the central bank moves after the public expect the inflation rate rule. In other words, given the anticipated inflation rate rule (3), the central bank chooses the optimum feedback rule (6). In a rational expectations equilibrium, however, the public and the central bank move simultaneously, so that the public cannot be surprised by the central bank. The consequences of this difference in timing assumption will be apparent in the next section.

With the simplifying assumption that $\bar{y}=0$, DGK analyzes the model consisting of equations (1),(2), and (6) under rational expectations, i.e.,(7). By the same calculation as that in DGK, the rational expectations equilibrium of this economy can be obtained in a more general case where $\bar{y} \geq 0$:

(8) $$A_1^{\#} = \frac{\alpha\lambda\bar{y}}{(1-\lambda)(1-\rho\beta)} + \pi^*, \quad A_2^{\#} = \frac{-\alpha\lambda\rho}{(1-\lambda)(1-\rho^2\beta)}.$$

Setting $\bar{y}=0$ (and fixing the relative weight on inflation variability in the objective function to unity, $1-\lambda=1$), (8) reduces to the rational expectations equilibrium obtained by DGK.

Next consider the optimal reaction function under commitment. Suppose that the central bank is committed ex ante to a particular feedback rule. In other words, commitment to particular values of its *feedback coefficients* $[A_1, A_2]$ is assumed to be possible. As the central bank is able to affect private agents' expectations of its feedback coefficients under commitment, it follows that

$$A_1^e = A_1, A_2^e = A_2.$$

Then we can write the constraint above as

(9) $$y_\tau = \rho y_{\tau-1} + \eta_\tau.$$

Substitute $y_\tau = \rho y_{\tau-1} + \eta_\tau$ and $\pi_\tau = A_1 + A_2 y_{\tau-1}$ for all $\tau \geq t$ into the intertemporal loss function (1) to obtain

$$(10) \quad L = \lambda[\frac{\beta\rho^2 y_{t-1}^2}{1-\beta\rho^2} - \frac{2\rho\bar{y}y_{t-1}}{1-\beta\rho} + \frac{\bar{y}^2}{1-\beta} + \frac{\sigma_\eta^2}{(1-\beta)(1-\beta\rho^2)}]$$

$$+(1-\lambda)[\frac{(A_1-\pi^*)^2}{1-\beta} + \frac{2(A_1-\pi^*)A_2 y_{t-1}}{1-\beta\rho} + \frac{A_2^2 y_{t-1}^2}{1-\beta\rho^2} + \frac{\beta A_2^2}{(1-\beta)(1-\beta\rho^2)}\sigma_\eta^2],$$

where σ_η^2 represents the variance of η_τ. Details of the solution procedure are presented in Section A of a companion working paper, Matsukawa Okamura and Taki [2007][4].

In what follows, it is convenient to decompose the central bank's loss function (10) into the following components:

$$(11) \quad L = \lambda L_Y[\bar{y}, y_{t-1}, \sigma_\eta^2, \beta, \rho] + (1-\lambda)L_\pi[y_{t-1}, \delta, A_2, \sigma_\eta^2, \beta, \rho]$$

$$= \lambda\{L_{DY}[\bar{y}, y_{t-1}, \beta, \rho] + L_{SY}[\beta, \rho]\sigma_\eta^2\}$$

$$+(1-\lambda)\{L_{D\pi}[\delta, y_{t-1}, A_2, \beta, \rho] + L_{S\pi}[A_2, \beta, \rho]\sigma_\eta^2\},$$

where $\delta = A_1 - \pi^*$ and each component is defined as

$$(12) \quad L_{DY}[\bar{y}, y_{t-1}, \beta, \rho] = \frac{\beta\rho^2 y_{t-1}^2}{1-\beta\rho^2} - \frac{2\rho\bar{y}y_{t-1}}{1-\beta\rho} + \frac{\bar{y}^2}{1-\beta}$$

$$(13) \quad L_{SY}[\beta, \rho]\sigma_\eta^2 = \frac{1}{(1-\beta)(1-\beta\rho^2)}\sigma_\eta^2$$

$$(14) \quad L_{D\pi}[\delta, y_{t-1}, A_2, \beta, \rho] = \frac{\delta^2}{1-\beta} + \frac{2\delta A_2 y_{t-1}}{1-\beta\rho} + \frac{A_2^2 y_{t-1}^2}{1-\beta\rho^2}$$

$$(15) \quad L_{S\pi}[A_2, \beta, \rho]\sigma_\eta^2 = \frac{\beta A_2^2}{(1-\beta)(1-\beta\rho^2)}\sigma_\eta^2.$$

Notice that the only component including A_1 is $L_{D\pi}[\delta, y_{t-1}, A_2, \beta, \rho]$, which depends on A_1 only through $\delta = A_1 - \pi^*$.

The central bank's problem is to minimize (10) with respect to $\delta(A_1)$ and A_2, where the minimization is subject to a given initial condition for y_{t-1}. Taking the results in BG into account, it is a reasonable conjecture that the feedback coefficients associated with the minimum of the central bank's loss function are $A_1 = \pi^*$ and $A_2 = 0$[5]. To prove that our conjecture is correct, it is enough to show that $L_\pi[y_{t-1}, \delta, A_2, \sigma_\eta^2, \beta, \rho]$ assumes one global minimum that corresponds to the BG optimum because only $L_\pi[y_{t-1}, \delta, A_2, \sigma_\eta^2, \beta, \rho]$ depends on $\delta(A_1)$ and A_2. The details of the proof are presented in Section B of Matsukawa Okamura and Taki [2007]. Here we summarize the results in the following proposition:

Proposition: Under commitment, the unique optimum rule is equivalent to a strict inflation targeting characterized by

(16) $\quad A_1^R = \pi^*, \quad A_2^R = 0,$

where the feedback coefficients of this optimum rule is denoted by (A_1^R, A_2^R).

Since monetary policy cannot affect long-run average output in a world consistent with the natural-rate hypothesis, it is reasonable that the optimum rule involves $A_1^R = \pi^*, A_2^R = 0$.

IV. Discretionary Equilibria under 'Active' and 'Passive' Monetary Policies

<u>The Central Bank's Best Response</u>

In the discretionary regime, the central bank chooses the optimum feedback rule given the public's expectations of feedback coefficients. The optimization problem is represented by a

Lagrangian of the form:

$$(17)\qquad \Lambda = E_t \sum_{\tau=t}^{\infty} \beta^{\tau} [\{\lambda(y_\tau - \bar{y})^2 + (1-\lambda)(\pi_\tau - \pi^*)^2\}$$

$$- \mu_\tau \{y_\tau + \alpha A_1^e - (\rho - \alpha A_2^e) y_{\tau-1} - \alpha \pi_\tau - \eta_\tau\}].$$

We differentiate the Lagrangian with respect to y_τ and π_τ to obtain the first-order conditions:

$$(18)\qquad 2\lambda(y_\tau - \bar{y}) - \mu_\tau + \beta(\rho - \alpha A_2^e) E_\tau \mu_{\tau+1} = 0,$$

$$(19)\qquad 2(1-\lambda)(\pi_\tau - \pi^*) + \alpha \mu_\tau = 0,$$

for each time τ. Conditions (18) and (19) imply the stochastic Euler equation:

$$(20)\qquad \lambda(y_\tau - \bar{y}) + \frac{(1-\lambda)(\pi_\tau - \pi^*)}{\alpha} - \frac{\beta(1-\lambda)(\rho - \alpha A_2^e)}{\alpha} E_\tau(\pi_{\tau+1} - \pi^*) = 0.$$

Substituting (6) and (3) for time $\tau+1$ into (20), and equating coefficients, we obtain values for A_1 and A_2 in terms of A_1^e, A_2^e, and parameters of the model. More concretely, setting to zero the coefficient on $y_{\tau-1}$ and the constant term leads to, respectively,

$$(21)\qquad A_2^2 - \frac{\alpha^2 \lambda + 1 - \lambda - \beta(1-\lambda)(\rho - \alpha A_2^e)^2}{\alpha\beta(1-\lambda)(\rho - \alpha A_2^e)} A_2 - \frac{\lambda}{\beta(1-\lambda)} = 0,$$

and

$$(22)\qquad A_1 = [\lambda\alpha^2 + (1-\lambda)\{1 - \beta(\rho - \alpha A_2^e)(1 + \alpha A_2)\}]^{-1}$$

$$[\lambda\alpha(\alpha A_1^e + \bar{y}) + (1-\lambda)\pi^* - \beta(1-\lambda)(\rho - \alpha A_2^e)(\alpha A_1^e A_2 + \pi^*)].$$

Section C of Matsukawa Okamura and Taki [2007] explains details of the derivations in this and in the next subsections.

We can use equations (21) and (22) to define the central bank's best response function

$(A_1,\ A_2) = B(A_1^e, A_2^e)$. In fact, these equations can be solved recursively to obtain the unique solution to the problem. First, find the negative root of the quadratic equation (21) in which A_1 is not included. As is shown in Section C of Matsukawa Okamura and Taki [2007], the quadratic equation (21) always has two real roots and the positive one is not relevant here. Therefore from now on we focus our attention to the negative root and denote it as $A_2 = f(A_2^e)$. Second, substituting A_2 obtained in this way into (22) yields the solution for A_1.

<u>Constructing Discretionary Equilibria</u>

We now turn to the characterization of a discretionary equilibrium. We show that there are two discretionary equilibria, one with deflation bias and one with high inflation bias. A necessary and sufficient condition for the central bank's feedback coefficients, $(A_1^D,\ A_2^D)$ to be chosen in a discretionary equilibrium is that $(A_1^D,\ A_2^D) = B(A_1^D, A_2^D)$. Setting $A_2^e = A_2 = A_2^D$ in (21), it follows:

$$(A_2^D)^2 - \frac{\alpha^2\lambda + 1 - \lambda - \beta(1-\lambda)(\rho - \alpha A_2^D)^2}{\alpha\beta(1-\lambda)(\rho - \alpha A_2^D)} A_2^D - \frac{\lambda}{\beta(1-\lambda)} = 0.$$

Rearranging terms yields:

(23) $$(1-\lambda)(A_2^D)^2 + \frac{(1-\lambda)(1-\beta\rho^2)}{\alpha\beta\rho} A_2^D + \frac{\lambda}{\beta} = 0.$$

Setting $A_2^e = A_2 = A_2^D$ and $A_1^e = A_1 = A_1^D$ in (22), and after rearranging, we have:

(24) $$A_1^D = \frac{\alpha\lambda\bar{y}}{(1-\lambda)\{1-\beta(\rho - \alpha A_2^D)\}} + \pi^*.$$

The discretionary equilibria of our model can be characterized by the central bank's feedback coefficients (A_1^D, A_2^D) satisfying the equations (23) and (24). It is important to note that the actual rate of inflation in a discretionary equilibrium, A_1^D can be lower than the optimum rate π^* when the absolute value of A_2^D is large enough to make $1-\beta(\rho-\alpha A_2^D)$ negative. In addition, if $\bar{y}>0$ is sufficiently large, or the central bank targets sufficiently higher level of output than the natural level, activist policy brings about deflation instead of high inflation in a discretionary equilibrium. We now have:

Theorem 1: Under discretion, remorseless activist monetary policy, which is very sensitive to changes in output gap, brings about a lower inflation rate than the optimum. Furthermore, if the central bank targets sufficiently higher level of output than the natural level, it can bring about deflation in a discretionary equilibrium.

Figure I illustrates the regions of ρ and λ where the roots of the quadratic equation (23) consist of real roots and of complex pairs for $\alpha = 0.5, 0.9$ and $\beta = .99$. The discriminant of (23) is negative (positive) for the set of (ρ, λ) located to the northeast (southwest) of the curves in the figure. From Figure I, we find that given ρ, as the weight put on output gap stabilization (λ) is increased, combinations of (ρ, λ) is more likely to make the discriminant of the quadratic equation (23) negative.

Figure I

<u>Two Types of Discretionary Equilibria</u>

Consider the case in which the discriminant of the quadratic equation (23) is negative and its roots consist of complex pairs. From Figure I, the higher the relative weight on output-gap stabilization, the less likely the best response function, (A_1, A_2)= $B(A_1^e, A_2^e)$ has fixed points. This implies that for λ close to unity it is always the best response for the central bank to choose more-active monetary policy than that anticipated by the public, i.e., $A_2 < A_2^e < 0$. In other words, if the central bank is mainly concerned with output-gap stabilization, the resulting monetary policy tends to be more and more active. In particular, when $\lambda = 1$, $A_2 = f(A_2^e)$ reduces to $A_2 = A_2^e - \dfrac{\rho}{\alpha}$. The present model puts no bound on A_2, although policies such as that setting an upper bound on public debt limit the value of A_2 in the real world.

Next, consider the case in which the discriminant of the quadratic equation (23) is positive and it has two distinct real roots, which are negative by Viète's formulas. Let A_2^{**} be one with greater absolute value and let A_2^* be the other with smaller absolute value. Denote the intercept associated with A_2^{**} and A_2^*, as A_1^{**} and A_1^*. In what follows, the discretionary equilibrium represented by (A_1^{**}, A_2^{**}) is called the "active equilibrium," and that represented by (A_1^*, A_2^*) is called the "passive equilibrium." We adopt this terminology from Leeper (1991), but there are two differences between our model and that found in the monetary rules literature. First, policy is usually referred to as active (passive) when the response of the nominal interest rate is more than (less than) one-for-one to inflation. Second, we do not restrict "passive policy" to the case that $|A_2^*| < 1$.

In the active equilibrium, A_2^{**} can be large enough (in absolute value) to make the denominator of (24) negative. Then for a positive target level of output gap, $\bar{y}$, A_1^{**} is lower than π^*. In particular, if the central bank tries to target an unrealistically high level of output, i.e., if $\bar{y}$ is large enough, the equilibrium rate of inflation under active monetary policy, A_1^{**} can be negative.

We now summarize the results in the following theorem:

Theorem 2:

(1) As long as the central bank does not place too much weight on output stability, there exist two discretionary equilibria. If the central bank tries to target an unrealistically high level of output, the resulting rate of inflation is higher (lower) than the social optimum in the active (passive) equilibrium.

(2) When too much weight is put on output gap stabilization by the central bank, the central bank's best response is to choose more-active policy than that expected by the public. A danger, here, of course, is that activist policies always become more and more sensitive to changes in output.

<u>Examples</u>

Given the values of the parameters, one can see that the equation (21) determines A_2 as a function of A_2^e alone. In Figures II-1 and II-2, the vertical axis represents A_2, the feedback coefficient on $y_{\tau-1}$ chosen by the central bank. The horizontal axis represents A_2^e, the value of A_2 expected by the public. As mentioned above, the quadratic equation (21) always has two real roots and the positive one (dashed line in these figures) is not relevant here. Therefore from now on we focus our attention to the negative root (solid line) denoted by $A_2 = f(A_2^e)$.

Figures II-1 and II-2

Using the parameter settings ($\alpha = .5$, $\beta = .99$, $\rho = .5$, $\lambda = .8$), Figure II-1 shows a graph of $A_2 = f(A_2^e)$. Since the quadratic equation (23) has no real root for this parameter configurations, it necessarily holds that $A_2 < A_2^e$. In contrast, Figure II-2 displays this function in our benchmark case --- $(\alpha = .5, \beta = .99, \rho = .5, \lambda = .5)$. It is seen that the quadratic equation (23) has two real roots for this parameter configurations and that the curve $A_2 = f(A_2^e)$ intersects the 45-degree line at $A_2^{**} = -2.6608$ (the active equilibrium) and $A_2^* = -0.37963$ (the passive equilibrium).

Table I shows the feedback coefficients chosen by the central bank in the active equilibrium for $(\alpha = 0.5, \beta = .99)$. In addition, given $\alpha = 0.5$, the bottom of Table I illustrates the results for $\beta = .96$ and $\rho = 0.5$. In this table, "No Discretionary Equilibrium" represents the case in which the parameter values used make the discriminant of (23) negative and the quadratic equation (23) has no real root.

The feedback coefficient, A_1^{**} depends on $\bar{y}$ and π^* as well as $\lambda, \alpha, \beta, \rho$ (see (24)). In spite of the fact that this table shows the results only for $\bar{y} = 0.01$ and $\pi^* = 0$, it is easy to obtain the feedback coefficient, A_1^{**} in the general case because (24) means that A_1^{**} is linear in $\bar{y}$ and π^*. For example, suppose that the optimum level of inflation is one percent ($\pi^* = 0.01$) and that the central bank desires to make the level of output four percent higher than the natural rate on average in our benchmark case--- $(\alpha = .5, \beta = .99, \rho = .5, \lambda = .5)$. Then in the active equilibrium, the rate of inflation chosen by the central bank observing $y_{t-1}=0$ is $A_1^{**}=0.01-0.0062\times 4=-0.0148$ (see row 13 column 1 of Table I). That is to say, in spite of the one percent target, the central bank chooses 1.48 percent rate of deflation in the active equilibrium. Interestingly, deflation, instead of inflation results in a discretionary equilibrium, in contrast to the argument of BG. Deflation arises because of $\beta(\rho-\alpha A_2^{**})>1$, so that the denominator of the expression $A_1^{**}=\dfrac{\alpha\lambda\bar{y}}{(1-\lambda)\{1-\beta(\rho-\alpha A_2^{**})\}}+\pi^*$ is negative. It follows immediately that the higher the level of output gap desired by the central bank, the higher the possibility that the deflationary equilibrium emerges. It is also clear from these tables that the possibility of deflation tends to be higher as the value of ρ increases.

Table I

We now turn to the passive equilibrium. Table II presents the feedback coefficients associated with the passive equilibrium for $\alpha=0.5$. Consider our benchmark case --- $\alpha=.5$, $\rho=0.5$, $\lambda=0.5$, $\beta=.99$, again. Since $A_1^*-\pi^*=0.01577$ for $\bar{y}=0.01$, if the optimum levels of output gap and inflation are $\bar{y}=0.04$ and $\pi^*=0.01$, respectively, the rate of inflation in the passive equilibrium will be $\pi=0.01+0.01577\times 4=0.07308$, i.e., 7.308 percent inflation (see row 13 column 1 of Table II). As in BG model, the resulting inflation rate is higher than the social optimum.

Table II

Figures III-1 and III-2 show the feedback coefficients associated with discretionary equilibria as functions of λ. We maintain the other parameters at the same values as in our benchmark calibration --- $\alpha = .5$, $\rho = 0.5$, $\beta = .99$.

Figures III-1 and III-2

According to Figure III-1, $A_1^{**} - \pi^*$ and hence A_1^{**} is a decreasing function of λ. This result implies that as the weight put on output gap stabilization is increased, monetary policy becomes less likely to be inflationary. Figure III-1 also shows that there is a discontinuity, $\tilde{\lambda}$ in the graph of $A_1 - \pi^*$, which corresponds to $\beta(\rho - \alpha A_2^*) = 1$ or $A_2^* = \dfrac{\beta\rho - 1}{\beta\alpha}$. In our benchmark calibration, $\tilde{\lambda} = 0.683$, which, together with $\alpha = .5$, $\rho = 0.5$ and $\beta = .99$ yields $A_2^* = -1.02$. As λ approaches to this critical value, which is well below unity, passive monetary policy generates a sharp acceleration of inflation. This result could explain the spike of US inflation in 1974-75.

Figure III-2 depicts the behaviors of A_2^{**} and A_2^* as functions of λ. Clearly, A_2^{**} is increasing in λ and A_2^* is decreasing in λ. The value of λ for which $A_2^{**} = A_2^*$, together with $\alpha = .5$, $\rho = 0.5$ and $\beta = .99$, yields the double root of the quadratic equation (23). In our benchmark case, this value of λ is about 0.701. Note that in its neighborhood, the absolute value of A_2^* exceeds one. In this respect, it is not always appropriate to call this policy "passive."

<u>Comparison to the Preceding Literatures and Intuition for the Active Equilibrium</u>

Since the BG static model used unemployment as the state variable, we replace unemployment with output gap and reformulate it as follows. The central bank minimizes a simple quadratic form: $E_\tau\{\lambda(y_\tau - \bar{y})^2 + (1-\lambda)(\pi_\tau - \pi^*)^2\}$ subject to $y_\tau = \alpha(\pi_\tau - \pi_\tau^e) + \eta_\tau$, where we use the notation of Section II. Then it is straightforward to show that the rate of inflation in the BG discretionary equilibrium is $A_1^{BG} = \dfrac{\alpha\lambda\bar{y}}{1-\lambda} + \pi^*$, which is higher than the optimum level π^*. The intuition behind this result is that in discretionary equilibrium, "*the expected rate of inflation is sufficiently high so that the marginal cost of inflation just balances the marginal gain from reducing unemployment*"(BG, p.599).

Now from (8), the rate of inflation in the rational expectations equilibrium is $A_1^\# = \dfrac{\alpha\lambda\bar{y}}{(1-\lambda)(1-\rho\beta)} + \pi^*$, which is apparently higher than the discretionary equilibrium of BG static model. In dynamic models, a positive output gap would lead to an increase in output in the following periods. Therefore, the marginal gain from positive output gaps is the expected discounted value of the sum of output increases in the DGK model. In order to balance the marginal cost with marginal gain, the expected rate of inflation must be raised. Next, let us compare the DGK rational expectations equilibrium with the discretionary equilibria obtained in this paper, $A_1^D = \dfrac{\alpha\lambda\bar{y}}{(1-\lambda)\{1-\beta(\rho-\alpha A_2^D)\}} + \pi^*$. The difference between these two rates is the term $\alpha\beta A_2^D$ in the denominator. Since the central bank minimizes the expected discounted losses given the feedback coefficients anticipated by the public in a discretionary equilibrium, the degree of output persistence faced by the central bank is $\rho - \alpha A_2^e$ instead of ρ. Therefore, $A_1^\#$ and A_1^D differ by $\alpha\beta A_2^D$ in the denominator.

If the central bank chooses a higher rate of inflation in the active equilibrium, where the rate of inflation is lower than the optimum rate, π^*, both the cost of inflation and output cost are decreased. Then it is a natural question to ask "why doesn't the central bank raise the rate of inflation in the active equilibrium?" To answer this question, we begin by studying the effects of a one-time deviation of inflation from an arbitrary feedback rule, $\pi_\tau = \widetilde{A}_1 + \widetilde{A}_2 y_{\tau-1}$. Ignoring the supply shock η_τ, and setting $y_0 = 0$, we suppose that in the initial period zero, the central bank chooses this feedback rule and that it is anticipated by the public: $A_1^e = \widetilde{A}_1$ and $A_2^e = \widetilde{A}_2$. In the first period, the central bank chooses another rate of inflation, $\pi_1 > \pi_0$ (or equivalently, $A_1(> \widetilde{A}_1)$ under the maintained assumption that $\eta_\tau \equiv 0$ and $y_0 = 0$), although the previous rule $\pi_\tau = \widetilde{A}_1 + \widetilde{A}_2 y_{\tau-1}$ continues to be in place and anticipated by the public in subsequent periods, $\tau \geq 2$ (Figure IV-1, upper panel).

Figure IV-1

The following paths of surprise inflation and output gap are depicted in the middle and lower panels of Figure IV-1. In the first period, the central bank conducts a surprise inflation policy: $\pi_1 - \pi_1^e = (A_1 + \widetilde{A}_2 y_0) - (A_1^e + A_2^e y_0) = A_1 - \widetilde{A}_1 > 0$ (Figure IV-1, middle panel). This policy produces a positive output gap, $y_1 = \rho y_0 + \alpha(\pi_1 - \pi_1^e) = \alpha(\pi_1 - \widetilde{A}_1) > 0$ in the same period. Since $\pi_\tau = \pi_\tau^e$ holds for $\tau \geq 2$, it follows that: $y_\tau = \rho y_{\tau-1} + \alpha(\pi_\tau - \pi_\tau^e) = \rho y_{\tau-1} > 0$, for $\tau \geq 2$. As the lower panel of Figure IV-1 shows, output gap approaches to zero from above, which keeps both actual and expected inflation rates below their original path. This is because $\pi_{\tau+1} = \pi_{\tau+1}^e = \widetilde{A}_1 + \widetilde{A}_2 y_\tau < \widetilde{A}_1 = \pi_0$ holds for $\tau \geq 2$.

We now introduce the notation $\Delta^\pi(\tau = 1)$, $\Delta^\pi(\tau \geq 2)$, Δ^π, and Δ^y, to refer to the marginal effects of a one-time deviation of inflation on π_1, π_τ for $\tau \geq 2$, π_τ for all $\tau \geq 1$, and y, respectively. Since $|y_\tau - \bar{y}| < |y_0 - \bar{y}| = \bar{y}$ for all periods, τ, the marginal output cost (marginal gain) of a one-time deviation of inflation is negative ($\Delta^y < 0$). Note that since the costs are evaluated by the quadratic functions, the marginal output cost (the marginal gain from positive output gaps, Δ^y) and the marginal costs of inflation ($\Delta^\pi(\tau = 1)$, $\Delta^\pi(\tau \geq 2)$, Δ^π) are proportional to the vertical distances $|y_\tau - \bar{y}|$ and $|\pi_\tau - \pi^*|$ indicated in Figures IV-1 and IV-2.

Figure IV-2

If $|\tilde{A}_2|$ is large enough, $|\pi_\tau - \pi_0| = |\tilde{A}_2 y_{\tau-1}|$ is significantly larger for $\tau \geq 2$ than $|\pi_1 - \pi_0|$ and the effect on π_1 ($\Delta^\pi(\tau = 1)$) is dominated by those on π_τ for $\tau \geq 2$ ($\Delta^\pi(\tau \geq 2)$) (see Figure IV-2)[6]. We now distinguish between two cases: an inflationary environment, where the actual rate of inflation is higher than the optimum rate ($\pi > \pi^*$), and a deflationary environment, where the actual rate of inflation is lower than the optimum rate ($\pi < \pi^*$).

First, consider a deflationary environment. Since $|\pi_1 - \pi^*| < |\pi_0 - \pi^*|$ for $\tau = 1$ and $|\pi_\tau - \pi^*| > |\pi_0 - \pi^*|$ for $\tau \geq 2$, it follows that $\Delta^\pi(\tau = 1) < 0$ and $\Delta^\pi(\tau \geq 2) > 0$ (see Figure IV-2, upper panel). Noting that $\Delta^\pi = \Delta^\pi(\tau = 1) + \Delta^\pi(\tau \geq 2)$, we conclude $\Delta^\pi > 0$ because $\Delta^\pi(\tau \geq 2) > 0$ is the dominating effect. If the initial value of $\tilde{A}_1 (= A_1^e)$ is far from π^*, $|\pi_\tau - \pi^*|$ is large relative to $|y_\tau - \bar{y}|$, and weighted sum of this marginal cost (Δ^π) and the marginal gain from positive output gaps (Δ^y) becomes positive: $\lambda\Delta^\pi + (1-\lambda)\Delta^y > 0$. If the initial value of $\tilde{A}_1 (= A_1^e)$ is close to π^*, $|\pi_1 - \pi^*|$ is small relative to $|y_\tau - \bar{y}|$, and weighted sum of this marginal cost and the marginal output gain becomes negative: $\lambda\Delta^{\pi}+(1-\lambda)\Delta^{y}<0$. The value of $\widetilde{A}_1$ for which this weighted sum becomes zero corresponds to the active equilibrium (A_2^{**}). These results are summarized in the upper panel of Table III.

Table III

Second consider an inflationary environment. Since $|\pi_1-\pi^*|>|\pi_0-\pi^*|$ for $\tau=1$ and $|\pi_\tau-\pi^*|<|\pi_0-\pi^*|$ for $\tau\geq 2$, it follows that $\Delta^{\pi}(\tau=1)>0$ and $\Delta^{\pi}(\tau\geq 2)<0$, implying that $\Delta^{\pi}<0$ because $\Delta^{\pi}(\tau\geq 2)$ is the dominating effect (see Figure IV-2, lower panel). Then weighted sum $\lambda\Delta^{\pi}+(1-\lambda)\Delta^{y}$ is also negative and we conclude that the active equilibrium cannot exist in an inflationary environment. This result is also presented in the upper panel of Table III.

If $|\widetilde{A}_2|$ is small enough, $|\pi_1-\pi_0|$ is significantly larger than $|\pi_\tau-\pi_0|=|\widetilde{A}_2 y_{\tau-1}|$ for $\tau\geq 2$, and the effect on π_1 ($\Delta^{\pi}(\tau=1)$) dominates that on π_τ for $\tau\geq 2$ ($\Delta^{\pi}(\tau\geq 2)$)[7]. Proceeding in the same manner, we can show that there exists a passive equilibrium only in an inflationary environment. The results for this case are summarized in Table III, lower panel.

Finally, from (8) the feedback coefficient on the lagged output gap chosen in the rational expectations equilibrium is $A_2^{\#}=\dfrac{-\alpha\lambda\rho}{(1-\lambda)(1-\rho^2\beta)}$. The absolute value of $A_2^{\#}$ is smaller than that of A_2^{*}, implying that the rational expectations equilibrium lies between the passive equilibrium and the social optimum level under commitment: $A_2^{*}<A_2^{\#}<0$[8].

<u>The Stability of Equilibria</u>

Differentiating (21) and (22) with respect to A_2^e and A_1^e, and evaluating them in the discretionary equlibria, we have: $\left.\frac{dA_2}{dA_2^e}\right|_{A_2^e=A_2^D} = -\frac{(1+\beta\rho^2)\alpha A_2^D}{(1-\beta^2\rho)(\rho-\alpha A_2^D)}$ and $\frac{\partial A_1}{\partial A_1^e} = \frac{\lambda\alpha^2 - \beta(1-\lambda)(\rho-\alpha A_2^D)\alpha A_2^D}{\lambda\alpha^2 - \beta(1-\lambda)(\rho-\alpha A_2^D)\alpha A_2^D + (1-\lambda)\{1-\beta(\rho-\alpha A_2^D)\}}$ (see Section D of Matsukawa Okamura and Taki [2007]).

Since $A_2^{**} < -\frac{1-\beta\rho^2}{2\alpha\beta\rho}$ for the active equilibrium, we have: $0 < (1-\beta^2\rho)(\rho-\alpha A_2^{**}) < -(1+\beta\rho^2)\alpha A_2^{**}$, and $\left.\frac{dA_2}{dA_2^e}\right|_{A_2=A_2^{**}} > 1$. For the passive equilibrium, it holds that: $A_2^* > -\frac{1-\beta\rho^2}{2\alpha\beta\rho}$. Then we obtain: $(1-\beta^2\rho)(\rho-\alpha A_2^*) > -(1+\beta\rho^2)\alpha A_2^*$, and $0 < \left.\frac{dA_2}{dA_2^e}\right|_{A_2=A_2^*} < 1$. The former result implies when the public anticipates more (less) active policy than that chosen in the active equilibrium, it is the best response for the central bank to choose even more (even less) active policy than anticipated by the public. The latter implies that when the public anticipates more (less) passive policy than that chosen in the passive equilibrium, the best response for the central bank is to choose less (more) passive policy than anticipated by the public. As an illustration, given a set of parameters, $\alpha = .5$, $\rho = 0.5$, $\lambda = 0.5$, $\beta = .99$, the graph of the central bank's best response (21) depicted in Figure II-2 cuts the diagonal from below (from above), as A_2^e approaches A_2^{**} (A_2^*) from below, implying that A_2^{**} (A_2^*) is unstable (stable).

It is also straightforward to show: $\left.\frac{\partial A_1}{\partial A_1^e}\right|_{A_1=A_1^{**}, A_2=A_2^{**}} > 1$, in the active equilibrium, and

$0 < \left.\frac{\partial A_1}{\partial A_1^e}\right|_{A_1=A_1^{**}, A_2=A_2^{**}} < 1$, in the passive equilibrium. The sequential nature of the solution allows us to conclude that the active (passive) equilibrium is unstable (stable). Note that the instability of the active equilibrium does not mean the instability of the dynamic system (5). In fact the degree of persistence at the active equilibrium is $\rho - \alpha A_2 - \alpha A_2^e = \rho$, so that the system is stable around this equilibrium.

<u>Inflation Targeting and Aggregate Demand Policy</u>

Our stability analysis exhibits inability of inflation targeting in eliminating a deflationary bias, when the economy is in the active equilibrium. Suppose that the economy has been in a discretionary equilibrium with deflationary bias. The central bank decides to change the target rate of inflation to eliminate bias. In the present model, this changing target should reflect the shift of the central bank's loss function, or changes in $\bar{y}$ or π^* [9].

In this model the feedback coefficient, A_1 depends on A_2, but the converse is not true. Furthermore, only A_1 depends on π^* or $\bar{y}$. Thus, even if the central bank raises the target level of inflation and in fact chooses a higher value of A_1, the value of A_2 chosen by the central bank remains the same as far as it faces the same Phillips curve. The following arguments hinge crucially on these properties of the model.

Suppose that the economy is in the active equilibrium and faced with deflation, and that the central bank adopts inflation targeting to raise the inflation rate and in fact chooses a higher value of A_1. As stated above, the optimum feedback coefficient A_2^{**} remains the same as far as the public continues to anticipate A_2^{**}. But in this situation, A_1^e does not always catch up with the increase in A_1 immediately, because the public tends to expect a lower rate of inflation than that is announced by the central bank: $A_1^e < A_1 < 0$. Since, $\frac{\partial A_1}{\partial A_1^e} > 1$ in the active equilibrium (see the previous subsection), the central bank's best response is to choose a lower rate of inflation than that expected by the public, so that such an inflation targeting regime will break down.

In contrast, if the economy is in the passive equilibrium, inflation targeting is effective in lowering inflation. When faced with high inflation, the central bank adopts inflation targeting to lower inflation. In this case, the public tends to expect a higher rate of inflation than that announced by the central bank. Since $0 < \frac{\partial A_1}{\partial A_1^e} < 1$ in the passive equilibrium, the central bank's best response is to choose lower inflation rate than anticipated by the public. Therefore, the economy approaches the new passive equilibrium and an inflation targeting regime can be supported by aggregate demand policy and succeeds in bringing inflation down.

In summary, we have:

Theorem 3: If inflation targeting is adopted to lower the rate of inflation, it can be supported by aggregate demand policies. In contrast, if it is adopted to escape from a deflationary trap, it cannot be supported by aggregate demand policies.

V. The Relative Welfare Performance of Alternative Regimes

The expressions for the central bank's loss function derived in Section III, (10) and (11), can also be used to evaluate expected losses in the stationary states associated with discretionary equilibria. This is because in discretionary equilibria, actual and privately-expected feedback rules coincide as in the social optimum under commitment $(A_1 = A_1^e, A_2 = A_2^e)$. Substituting $(A_1^{**},\ A_2^{**})$ and $(A_1^{*},\ A_2^{*})$ into (11), and after taking $\delta = A_1 - \pi^*$ into account, we obtain the implied value for each component of loss function.

In order to analyze the performance of alternative policy regimes, it is convenient to set initial value y_{t-1} to zero and to compare the resulting inflation variability and output variability. Define $L_x(A^{**}), L_x(A^{*})$ and $L_x(A^R)$ as the implied values of the components for the active equilibrium, for the passive equilibrium, and for the social optimum under commitment respectively, where we use the subscripts to distinguish among the six kinds of components, $x = SY, S\pi, S, DY, D\pi,$ and D.

First, setting $y_{t-1} = 0$, consider the deterministic components of the central bank's loss function, $L_{DY} = \dfrac{\bar{y}^2}{1-\beta}$, and $L_{D\pi} = \dfrac{(A_1 - \pi^*)^2}{1-\beta}$. In discretionary equilibria, privately-expected feedback coefficients coincide with those chosen by the central bank, which is exactly the condition that monetary policy cannot affect long-run average output in a world consistent with the natural-rate hypothesis. This is the reason that neglecting the effect of initial value, y_{t-1} makes L_{DY} independent of the underlying parameters except for β. Next, it is clear that $L_{D\pi}(A^R) = 0$ and that for discretionary equilibria $L_{D\pi} = \dfrac{(A_1 - \pi^*)^2}{1-\beta} = \dfrac{1}{1-\beta}[\dfrac{\alpha\lambda\bar{y}}{(1-\lambda)\{1-\beta(\rho - \alpha A_2^D)\}}]^2$. Note that $L_{D\pi}(A^{**})$ and $L_{D\pi}(A^{*})$ depend on $\bar{y}$ as well as the underlying parameters, but not on π^{*}. Since $L_{DY} = \dfrac{\bar{y}^2}{1-\beta}$ is independent of feedback coefficients, it is clear that $L_{DY}(A^{**}) = L_{DY}(A^{*}) = L_{DY}(A^{R})$. In contrast, as shown in Section F of Matsukawa Okamura and Taki [2007], we have: $L_{D\pi}(A^{R}) < L_{D\pi}(A^{**}) < L_{D\pi}(A^{*})$, where the exception is the case in which ρ is large enough to satisfy $\rho > \dfrac{1-\sqrt{1-\beta}}{\beta}$. Note that the deterministic component of inflation variability for the passive equilibrium, $L_{D\pi}(A^{*})$ grows rapidly, as λ approaches a critical value, $\tilde{\lambda}$ defined in the previous section.

Second, substituting $y_{t-1} = 0$ into L_{SY} and $L_{S\pi}$ obtains $L_{SY}[\beta, \rho] = \dfrac{1}{(1-\beta)(1-\beta\rho^2)}$ and $L_{S\pi}[A_2, \beta, \rho] = \dfrac{\beta A_2^2}{(1-\beta)(1-\beta\rho^2)}$. The stochastic component of output variation, L_{SY}, or the coefficient on σ_η^2 in the variance in output gap is independent of λ, and the three alternative regimes yield the same value of L_{SY} because of the same reason as stated above. In contrast, the stochastic component of inflation variability, or more precisely its coefficient on σ_η^2 ($L_{S\pi}$), differs among three alternative regimes. Since $0 = |A_2^R| < |A_2^*| < |A_2^{**}|$, the following conclusions follow: $0 = L_{S\pi}(A^{R}) < L_{S\pi}(A^{*}) < L_{S\pi}(A^{**})$.

Now both output variability and inflation variability are taken into account. Of course, the optimum under commitment (strict inflation targeting) generates the best results because the optimum under commitment minimizes the central bank's loss function subject to $A_1 = A_1^e$ and $A_2 = A_2^e$, the conditions that are also satisfied in discretionary equilibria. Since the three alternative regimes yield the same values of L_{DY} and L_{SY}, it is also clear that the behaviors of L_D and L_S are governed by those of $L_{D\pi}$ and $L_{S\pi}$. Then we have: $L_S(A^R) < L_S(A^*) < L_S(A^{**})$ and $L_D(A^R) < L_D(A^{**}) < L_D(A^*)$. In words, with respect to the deterministic (stochastic) components, the active equilibrium is superior (inferior) to the passive equilibrium. Again, the exception to the conclusion about the deterministic components is the case in which ρ is large enough to satisfy $\rho > \frac{1-\sqrt{1-\beta}}{\beta}$. These results are summarized in Table IV.

Table IV

Maintaining the assumption that $y_{t-1} = 0$, we now turn to the analysis of the total loss, $L = L_D + L_S\sigma_\eta^2$. First, the deterministic component, L_D is proportional to the square of $\bar{y}$, under this assumption. Second, the stochastic component is linear in σ_η^2. Then the welfare performance of alternative equilibria depends on the relative importance of σ_η^2 and $\bar{y}$. For most values of λ, the passive equilibrium is better than the active equilibrium if σ_η^2 is more important than $\bar{y}$, and vice versa. This result can be obtained from Table IV ⑤ and ⑥ immediately. For λ close to $\tilde{\lambda}$, even if σ_η^2 is more important than $\bar{y}$, the active equilibrium is superior to the passive equilibrium because $L_{D\pi}$ becomes infinite as λ approaches to $\tilde{\lambda}$.

The relative welfare performance of alternative monetary policy regimes will be summarized as follows:

Theorem 4: The social optimum under commitment (strict inflation targeting) yields the minimum loss. Passive monetary policy provides smaller losses than does active monetary policy over a wide range of parameter magnitudes if supply shocks are the dominant source of fluctuation. In contrast, active monetary policy is better if the central bank tries to target an unrealistically high level of output.

The policy implication of Theorem 4 is important. In the late 1960s and the 1970s, activist monetary policy tried to target an unrealistically high level of output. Therefore, passive monetary policy led to high inflation and significant welfare losses. Active monetary policy could have delivered better results and the rules under commitment (the strict inflation targeting) should have achieved the best. It is true that many countries have succeeded in maintaining low inflation in recent years. However, this does not always imply switching from passive monetary policy to the optimum under commitment (strict inflation targeting). Rather, it may reflect switching to active monetary policy. Then Theorem 4 tells us that the regime change from the passive to the active is successful if and only if the central bank still targets an unrealistically high level of output. If the central bank taking the natural rate hypothesis fully into consideration has not targeted an unrealistically high level of output, passive monetary policy regime is better than the active monetary policy regime. In this situation supply shocks are the major source of fluctuation, and therefore such passive monetary policy regime as "fine-tuning" produces the smaller expected loss.

Given $\bar{y}=0.01$ and various values of the parameters α, β, ρ, λ, Tables I and II also report some representative results for L_{DY}, $L_{D\pi}$, L_{SY} and $L_{S\pi}$ (columns 3 through 6) along with weighted averages of these components with weight, λ and $1-\lambda$, L_D and L_S (columns 7 and 8). Table I is for the active equilibrium and Table II is for the passive equilibrium. Note that L_{SY}, $L_{S\pi}$ and L_S represent the coefficients on σ_η^2 (the variance of the shocks, η_τ) of each stochastic component. Assuming that $\sigma_\eta^2 = 0.0001$ and $\bar{y} = 0.01$, we also express the minimum expected discounted loss, L, as the sum of $L_S\sigma_\eta^2$ and L_D in column 9.

Figures V-1 and V-2 depict the stochastic and deterministic components of inflation variability as functions of λ. We continue to take $\alpha = .5$, $\rho = 0.5$, $\beta = .99$ as our benchmark case. Figure V-1 shows that $L_{S\pi}[A_2, \beta, \rho]$ is an increasing (a decreasing) function of λ for the active (passive) equilibrium, whereas Figure V-2 indicates $L_{D\pi}[A_1, A_2, y_{t-1}, \beta, \rho]$ is an increasing function for both active and passive equilibria. The results presented in Table IV- ③ and ④ are easily confirmed from these figures.

Figures V-1, V-2

Maintaining the assumption that $y_{t-1} = 0$, Figures V-3 and V-4 illustrate the total loss, $L = L_D + L_S\sigma_\eta^2$ as functions of λ. With respect to the passive equilibrium, however, the deterministic component of inflation variability, $L_{D\pi}$, and hence the total loss, L is explosive in the neighborhood around the critical value, $\tilde{\lambda}$ (see Section IV, Figure III-1). Therefore in these figures the total loss, L is shown only for λ bounded away from $\tilde{\lambda}$. Fixing $\bar{y} = 0.01$ and $\pi^* = 0.01$, the value of σ_η^2 is set to 0.01 in Figure V-3 and to 0.000001 in Figure V-4. From these figures it is seen that the passive equilibrium is superior (inferior) to the active equilibrium when σ_η^2 is relatively more (less) important than $\bar{y}$.

Figures V-3, V-4

VI. Conclusion and Discussions

In this paper, we developed a dynamic version of BG model under which the dynamics of the model is driven by the persistency in output gap. The model reveals an interesting implication about activist monetary policies under the BG discretionary regime: remorseless activist may produce deflation instead of high inflation. We make use of game-theoretic constructs to discuss the interaction between the expectations formation of the public and the central bank's choice of feedback rule. The contrast between the expectations formation process and the relative timing of moves assumed in this model and those assumed in the previous literature is essential for this surprising result. Our main conclusions follow:

1. Except for the case of the double root, there are two (if any) discretionary equilibria. The resulting two discretionary equilibria are characterized as a deflationary and a high-inflationary equilibrium. We call the former the active equilibrium and refer to the associated monetary policy as active. The central bank's feedback rule chosen in the active equilibrium has a larger (in absolute value) coefficient on the lagged output gap. The latter is called the passive equilibrium and the associated monetary policy is referred to as passive. In Theorem 2, we showed that the "active" equilibrium may result in the deflationary one.
2. If the weight put on output gap stabilization by the central bank is large, it is always the best

response for the central bank to choose more-active monetary policy than that anticipated by the public. In other words, if the central bank is mainly concerned with output-gap stabilization, the resulting monetary policy tends to be more and more active, or more and more sensitive to changes in output gap. The present model puts no bound on the value of feedback coefficient on the lagged output gap, although policies such as that setting an upper bound on public debt limit the value of it in the real world.

3. When the public anticipates more (less) active policy than that chosen in the active equilibrium, it is the best response for the central bank to choose even more (even less) active policy than anticipated by the public. In contrast, when the public anticipates more (less) passive policy than that chosen in the passive equilibrium, the best response for the central bank is to choose less (more) passive policy than anticipated by the public. In other words, the active equilibrium is unstable and the passive equilibrium is stable.

4. Inflation targeting has asymmetric effects, depending on whether it is adopted to lower the rate of inflation or to escape from a deflationary trap. In the former case, it is effective in bringing inflation down because it can be supported by aggregate demand policies. In the latter case, however, it cannot be supported by aggregate demand policies and is not effective. Inflation targeting in this context differs from strict inflation targeting (the optimum rule under commitment).

5. Under commitment, the optimum rule is unique and it would reduce to a strict inflation targeting rule whereby the inflation rate is kept at its optimum level at all times. Since

monetary policy cannot affect long-run average output in a world consistent with the natural-rate hypothesis, it is reasonable that the optimum rule involves zero coefficient on the lagged output gap. The optimum rule under commitment is time-inconsistent.

6. The welfare consequences of alternative monetary policy regimes will be summarized as follows: Taking both output variability and inflation variability into account, the optimum under commitment (strict inflation targeting) yields the minimum loss. Passive monetary policy provides smaller losses than does active monetary policy over a wide range of parameter magnitudes if supply shocks are the dominant source of fluctuation. In contrast, active monetary policy is better if the central bank tries to target an unrealistically high level of output, as was observed in the late 1960s and the 1970s.

References

Barro, Robert, J., and David B. Gordon, 1983, "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy* 91, 589-610.

Benhabib, Jess, S. Schmitt-Grohe and M. Uribe, 2002, "Avoiding Liquidity Traps," *Journal of Political Economy,* 110, pp.535-563.

Chow, Gregory, 1981, *Econometric Analysis by Control Methods.* New York: Wiley.

Clarida, Richard, J. Gali, and M. Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115, pp. 147-180.

Dittmar, Robert and William T. Gavin, 2000, "What Do New-Keynesian Phillips

Curves Imply for Price-Level Targeting?" *Economic Review of the Federal Reserve Bank of St. Louis* (April 2000), pp. 21-30.

Dittmar, Robert, William T. Gavin and Finn Kydland, 1999, "The Inflation-Output Variability Trade-off and Price-Level Targeting," *Economic Review of the Federal Reserve Bank of St. Louis*, pp. 23-39.

Judd, John P., and Glenn D. Rudebusch (1998), "Taylor's Rule and the Fed: 1970–1997," Federal Reserve Bank of San Francisco Economic Review 1998(3), 3–16.

King, Robert G. and Alexander L. Wolman, 2004, "Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria," *Quarterly Journal of Economics,* Vol. 119 (4) pp. 1513-53.

Leeper, Eric M., "Equilibrium under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics,* Vol.27 (February 1991): 129-47.

Okamura, Kumiko, Shigeru Matsukawa, and Atsuhiro Taki, 2006, "Monetary Policy during the Deflation Period of the 1990s: Is Inflation Targeting Effective?" mimeo, (in Japanese).

Svensson, Lars E.O., 1997, "Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts," American Economic Review 87 (March 1997), pp. 98-114.

Svensson, Lars E.O., 1999, "Price-Level Targeting vs. Inflation Targeting: A Free Lunch?" *Journal of Money, Credit, and Banking* 31, 277-295.

Taylor, John B, 1993. "Discretion versus Rules in Practice." *Carnegie-Rochester*

Conference Series Public Policy 39 (December 1993), 195-214.

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Footnotes

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1 The model is obviously an extremely simple one, and in particular, the absence of a nominal interest rate reaction function is not entirely realistic.

2 Benhabib, Schmitt-Grohe and Uribe [2002, p.536] pointed out two important elements common across recent studies on monetary rules: first, they restrict attention to local dynamics, or small fluctuations, around a target level of inflation; and second, they do not take into account the fact that nominal interest rates are bounded below by zero. Benhabib, Schmitt-Grohe and Uribe [2002, p.536] concluded that these two simplifications have serious consequences for aggregate stability. Interestingly, we have arrived at similar results following a different route.

3 DGK specifies the information set of the central bank, which differs from that of the private sector. They assume that current realizations of supply shocks are in the information set of the central bank at the time it makes policy decisions, but not in that of the private sector (see Okamura Matsukawa and Taki [2006]). In the present model, however, the central bank's information set does not include current realizations of supply shocks.

4 Matsukawa Okamura and Taki [2007] is available on request.

5 If current realizations of supply shocks are in the information set of the central bank but not in that of the private sector, monetary policy is effective to the extent that the feedback coefficient on supply shocks have non-zero feedback coefficient in the central bank's reaction function (see Okamura Matsukawa and Taki [2006]).

6 For example, $\left|A_2^{**}\right| = 2.66078$ in the active equilibrium for our benchmark case…

$(\alpha = .5, \beta = .99, \rho = .5, \lambda = .5)$. See Table 1, column 2, row 13.

7 For example, $|A_2^*| = 0.37963$ in the passive equilibrium for our benchmark case···

$(\alpha = .5, \beta = .99, \rho = .5, \lambda = .5)$. See Table 2, column 2, row 13.

8 The social optimum level, A_2^R is, of course, zero.

9 Notice that unlike the case of strict inflation targeting (the optimum with commitment), the target rate of inflation in the present context is not equal to the optimum rate of inflation, π^*, as far as $\bar{y}$ differs from zero.

Table I The Active Equilibrium

$\alpha = 0.5, \beta = 0.99, 0.96$

β	ρ	λ	$A_1 - \pi^*$	A_2	L_{DY}	$L_{D\pi}$	L_{SY}	$L_{S\pi}$	L_D	L_S	L
0.99	0.1	0	0.00000	-20.00202	0.01000	0.00000	100.99990	40004.04	0.01000	40105.04	4.02050
0.99	0.1	0.3	-0.00024	-19.98035	0.01000	0.00001	100.99990	39917.41	0.01001	40018.41	4.01185
0.99	0.1	0.5	-0.00056	-19.95139	0.01000	0.00003	100.99990	39801.78	0.01003	39902.78	4.00030
0.99	0.1	0.7	-0.00130	-19.88348	0.01000	0.00017	100.99990	39531.28	0.01017	39632.28	3.97340
0.99	0.1	0.9	-0.00513	-19.53670	0.01000	0.00263	100.99990	38164.41	0.01263	38265.41	3.83917
0.99	0.3	0	0.00000	-6.13401	0.01000	0.00000	109.78153	4089.34	0.01000	4199.12	0.42991
0.99	0.3	0.3	-0.00093	-6.06260	0.01000	0.00009	109.78153	3994.68	0.01009	4104.46	0.42053
0.99	0.3	0.5	-0.00222	-5.96466	0.01000	0.00049	109.78153	3866.66	0.01049	3976.44	0.40814
0.99	0.3	0.7	-0.00548	-5.72211	0.01000	0.00300	109.78153	3558.58	0.01300	3668.36	0.37984
0.99	0.3	0.9	-0.04116	-3.62879	0.01000	0.16943	109.78153	1431.16	0.17943	1540.94	0.33352
0.99	0.5	0	0.00000	-3.04040	0.01000	0.00000	132.89037	1216.16	0.01000	1349.05	0.14491
0.99	0.5	0.3	-0.00231	-2.89064	0.01000	0.00054	132.89037	1099.30	0.01054	1232.19	0.13755
0.99	0.5	0.5	-0.00616	-2.66078	0.01000	0.00379	132.89037	931.42	0.01379	1064.31	0.12022
0.99	0.5	.7-.9	No Discretionary Equilibrium					No Discretionary Equilibrium			
0.99	0.7	0	0.00000	-1.48600	0.01000	0.00000	194.21247	424.57	0.01000	618.78	0.07188
0.99	0.7	0.3	-0.00925	-1.08818	0.01000	0.00856	194.21247	227.67	0.01856	421.89	0.06075
0.99	0.7	.5-.9	No Discretionary Equilibrium					No Discretionary Equilibrium			
0.99	0.9	0	0.00000	-0.44467	0.01000	0.00000	504.79556	98.82	0.01000	603.61	0.07036
0.99	0.9	.3-.9	No Discretionary Equilibrium					No Discretionary Equilibrium			
0.96	0.5	0	0.00000	-3.16667	0.00250	0.00000	32.89474	316.67	0.00250	349.56	0.03746
0.96	0.5	0.3	-0.00231	-3.01878	0.00250	0.00013	32.89474	287.78	0.00263	320.67	0.0347
0.96	0.5	0.5	-0.00609	-2.79382	0.00250	0.00093	32.89474	246.49	0.00343	279.38	0.03137
0.96	0.5	0.7	-0.03131	-1.85972	0.00250	0.02450	32.89474	109.22	0.02700	142.11	0.04121
0.96	0.5	0.9	No Discretionary Equilibrium					No Discretionary Equilibrium			

Table II The Passive Equilibrium

$\alpha = 0.5, \beta = 0.99, 0.96$

β	ρ	λ	$A_1 - \pi^*$	A_2	L_{DY}	$L_{D\pi}$	L_{SY}	$L_{S\pi}$	L_D	L_S	L
0.99	0.1	0	0	0	0.01	0	100.9999	0	0.01	100.9999	0.02010
0.99	0.1	0.3	0.00241	-0.02167	0.01	0.000579	100.9999	0.046954	0.010579	101.0469	0.02068
0.99	0.1	0.5	0.00571	-0.05063	0.01	0.003258	100.9999	0.256314	0.013258	101.2562	0.02338
0.99	0.1	0.7	0.01385	-0.11854	0.01	0.019184	100.9999	1.405031	0.029184	102.4049	0.03942
0.99	0.1	0.9	0.0671	-0.46532	0.01	0.450206	100.9999	21.65008	0.460206	122.65	0.47247
0.99	0.3	0	0	0	0.01	0	109.7815	0	0.01	109.7815	0.02098
0.99	0.3	0.3	0.00321	-0.07141	0.01	0.00103	109.7815	0.55422	0.01103	110.3358	0.02206
0.99	0.3	0.5	0.00808	-0.16935	0.01	0.006521	109.7815	3.116986	0.016521	112.8985	0.02781
0.99	0.3	0.7	0.02337	-0.41189	0.01	0.054638	109.7815	18.43856	0.064638	128.2201	0.07746
0.99	0.3	-0.9	0.08379	-2.50522	0.01	0.001945	109.7815	682.1129	0.011945	791.8944	0.09113
0.99	0.5	0	0	0	0.01	0	132.8904	0	0.01	132.8904	0.02329
0.99	0.5	0.3	0.00497	-0.14976	0.01	0.002473	132.8904	2.950668	0.012473	135.8410	0.02606
0.99	0.5	0.5	0.01577	-0.37963	0.01	0.024865	132.8904	18.9605	0.034865	151.8509	0.05005
0.99	0.5	.7-.9		No Discretionary Equilibrium					No Discretionary Equilibrium		
0.99	0.7	0	0	0	0.01	0	194.2125	0	0.01	194.2125	0.02942
0.99	0.7	0.3	0.01947	-0.39782	0.01	0.037895	194.2125	30.42885	0.047895	224.6413	0.07036
0.99	0.7	.5-.9		No Discretionary Equilibrium					No Discretionary Equilibrium		
0.99	0.9	0	0	0	0.01	0	504.7956	0	0.01	504.7956	0.06048
0.99	0.9	.3-.9		No Discretionary Equilibrium					No Discretionary Equilibrium		
0.96	0.5	0	0	0	0.0025	0	32.89474	0	0.0025	32.89474	0.00579
0.96	0.5	0.3	0.00477	-0.14788	0.0025	0.000569	32.89474	0.690584	0.003069	33.58532	0.00643
0.96	0.5	0.5	0.01466	-0.37285	0.0025	0.005374	32.89474	4.390014	0.007874	37.28475	0.01160
0.96	0.5	0.7	0.10869	-1.30695	0.0025	0.295354	32.89474	53.94058	0.297854	86.83531	0.30654
0.96	0.5	0.9		No Discretionary Equilibrium					No Discretionary Equilibrium		

Table III: The Effects of one-time deviation of inflation from a fixed path

Case 1: $|\tilde{A}_2|$ is large

		$\Delta^{\pi}(\tau=1)$	$\Delta^{\pi}(\tau\geq 2)$	Δ^{π}	Δ^{y}	$\lambda\Delta^{y}+(1-\lambda)\Delta^{\pi}$
I		+	−	−	−	−
D	$\tilde{A}_1$ close to π^*	−	+	+	−	−
	$\tilde{A}_1 = A_1^{**}$	−	+	+	−	0
	$\tilde{A}_1$ far from π^*	−	+	+	−	+

Case 2: $|\tilde{A}_2|$ is small

		$\Delta^{\pi}(\tau=1)$	$\Delta^{\pi}(\tau\geq 2)$	Δ^{π}	Δ^{y}	$\lambda\Delta^{y}+(1-\lambda)\Delta^{\pi}$
I	$\tilde{A}_1$ close to π^*	+	−	+	−	−
	$\tilde{A}_1 = A_1^{*}$	+	−	+	−	0
	$\tilde{A}_1$ far from π^*	+	−	+	−	+
D		−	+	−	−	−

Note I: Inflationary Environment, D: Deflationary Environment.

Table IV: The Deterministic and Stochastic Components of the Loss Function

	The stochastic components	The deterministic components
Output variability	① $L_{SY}(A^{**}) = L_{SY}(A^{*}) = L_{SY}(A^{R})$	② $L_{DY}(A^{**}) = L_{DY}(A^{*}) = L_{DY}(A^{R})$
Inflation variability	③ $L_{S\pi}(A^{R}) < L_{S\pi}(A^{*}) < L_{S\pi}(A^{**})$	④ $L_{D\pi}(A^{R}) < L_{D\pi}(A^{**}) < L_{D\pi}(A^{*})$
The weighted averages	⑤ $L_{S}(A^{R}) < L_{S}(A^{*}) < L_{S}(A^{**})$	⑥ $L_{D}(A^{R}) < L_{D}(A^{**}) < L_{D}(A^{*})$

Note that there exist exceptions to ④ and hence ⑥ (see Appendix E).

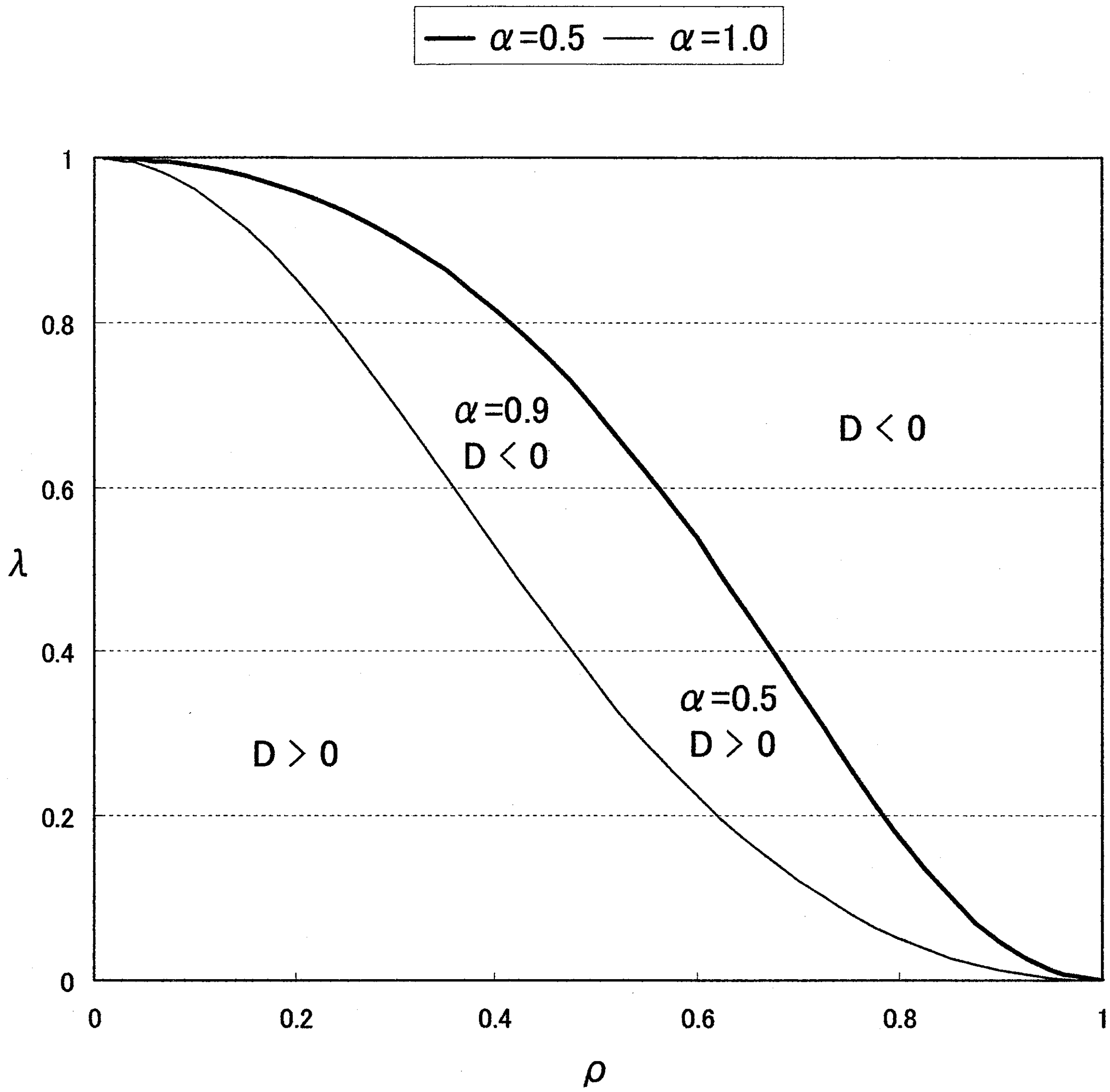


Figure I The Discriminant of Equation (23)
β=.99

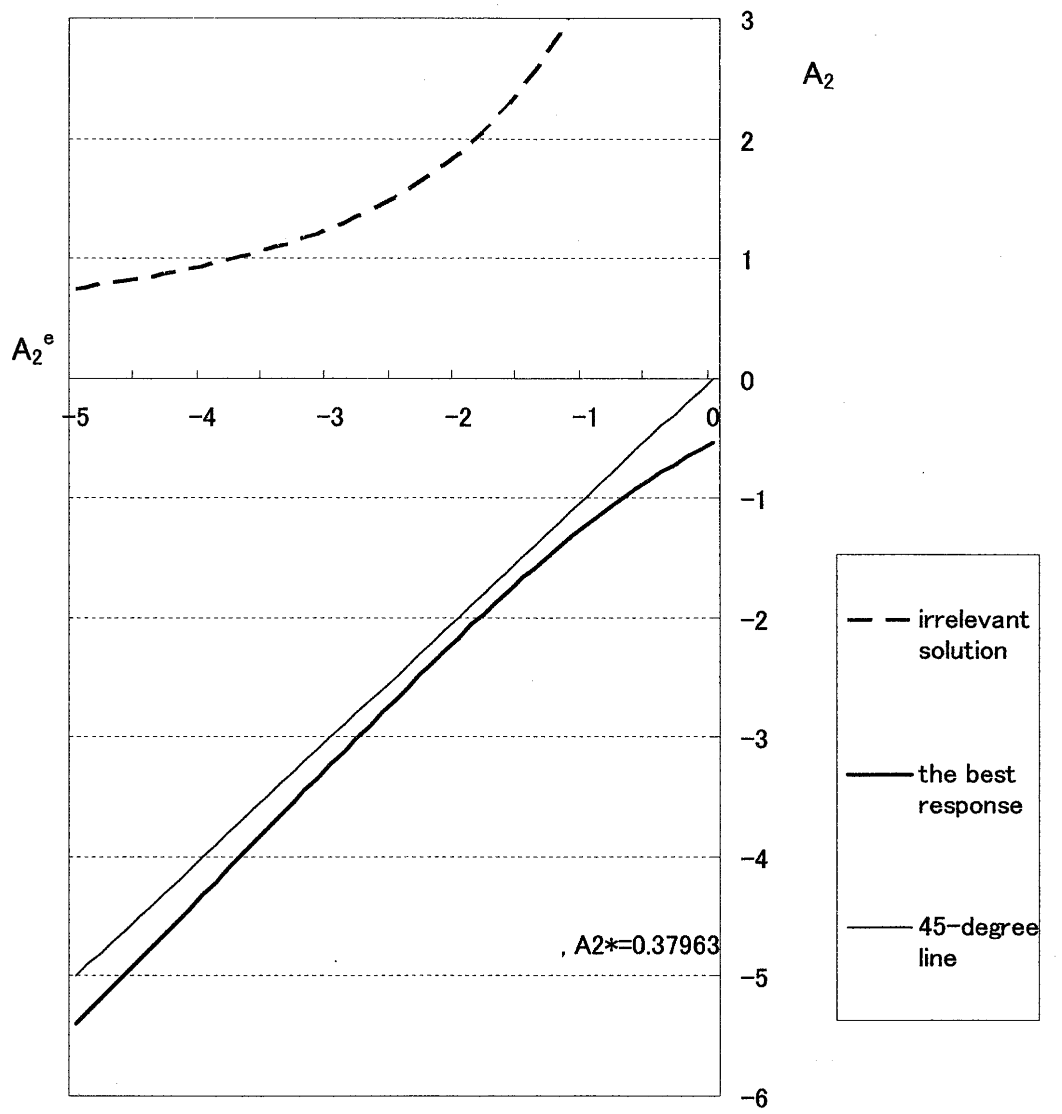


Figure II-1 Non Exsistence of Discretionary Equilibrium
α=0.5, β=.99, ρ=0.5, λ=0.8

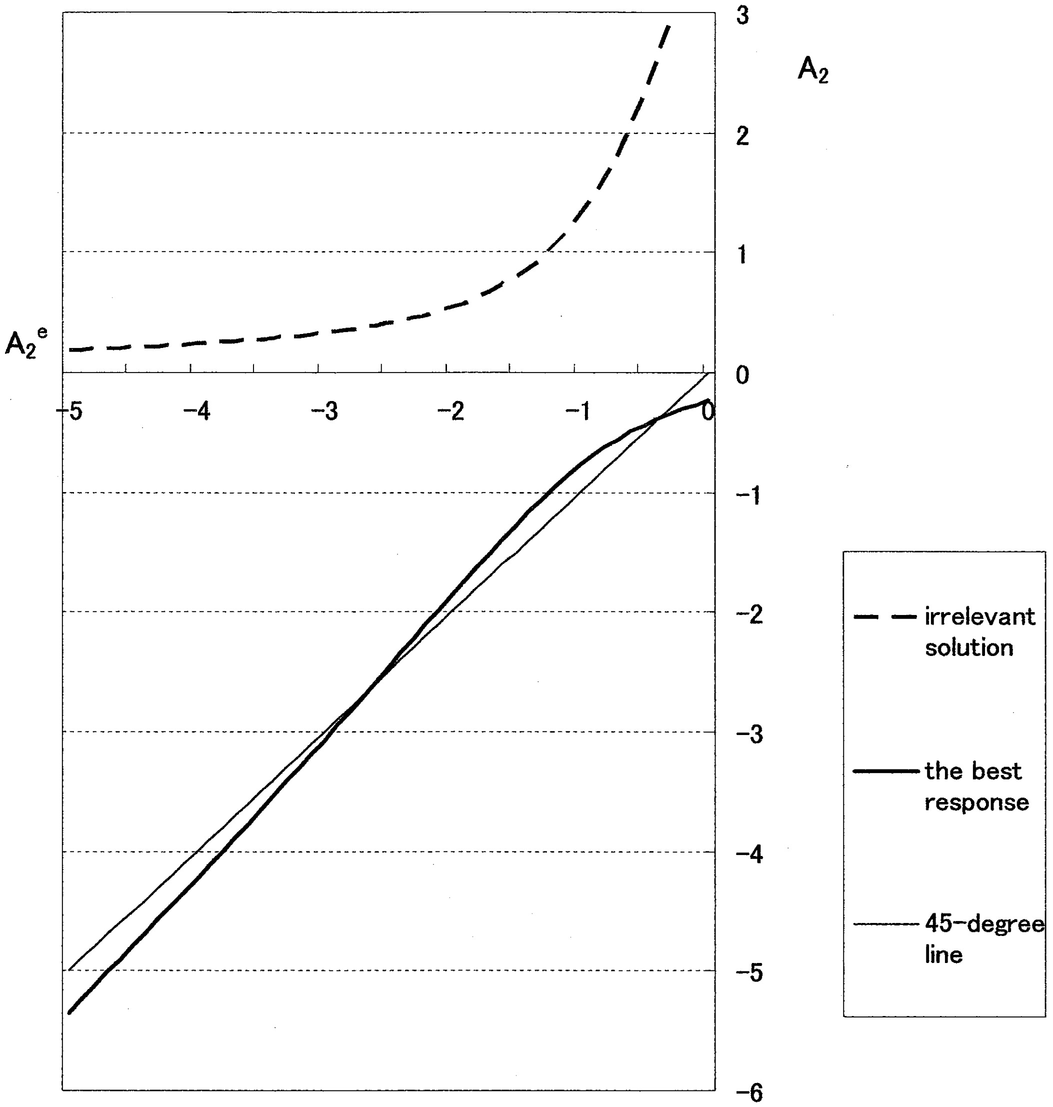


Figure II-2 Discretionary Equilibrium

A_2^{**}=-2.6609, A_2^{*}=-0.37963

α=0.5, β=.99, ρ=0.5, λ=0.5

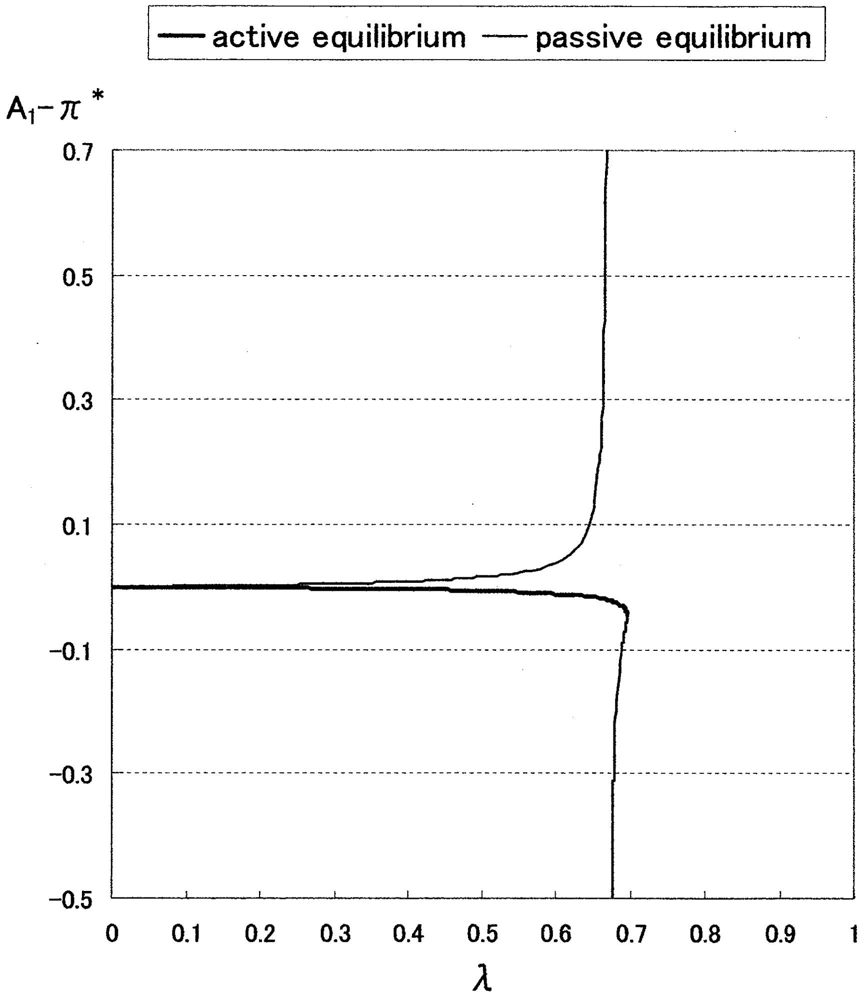


Figure III-1 Discretionary Equilibria (A_1)
α=0.5, ρ=0.5, β=.99

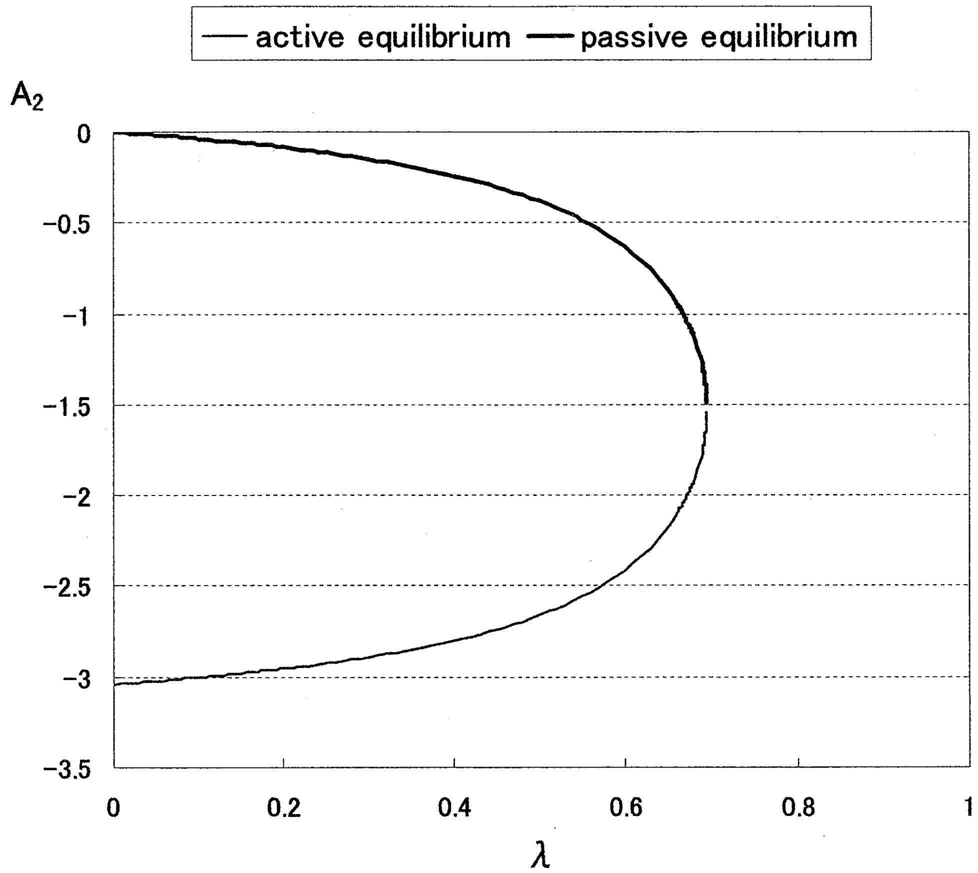


Figure III-2 Discretionary Equilibria (A_2)
α=0.5, ρ=0.5, β=.99

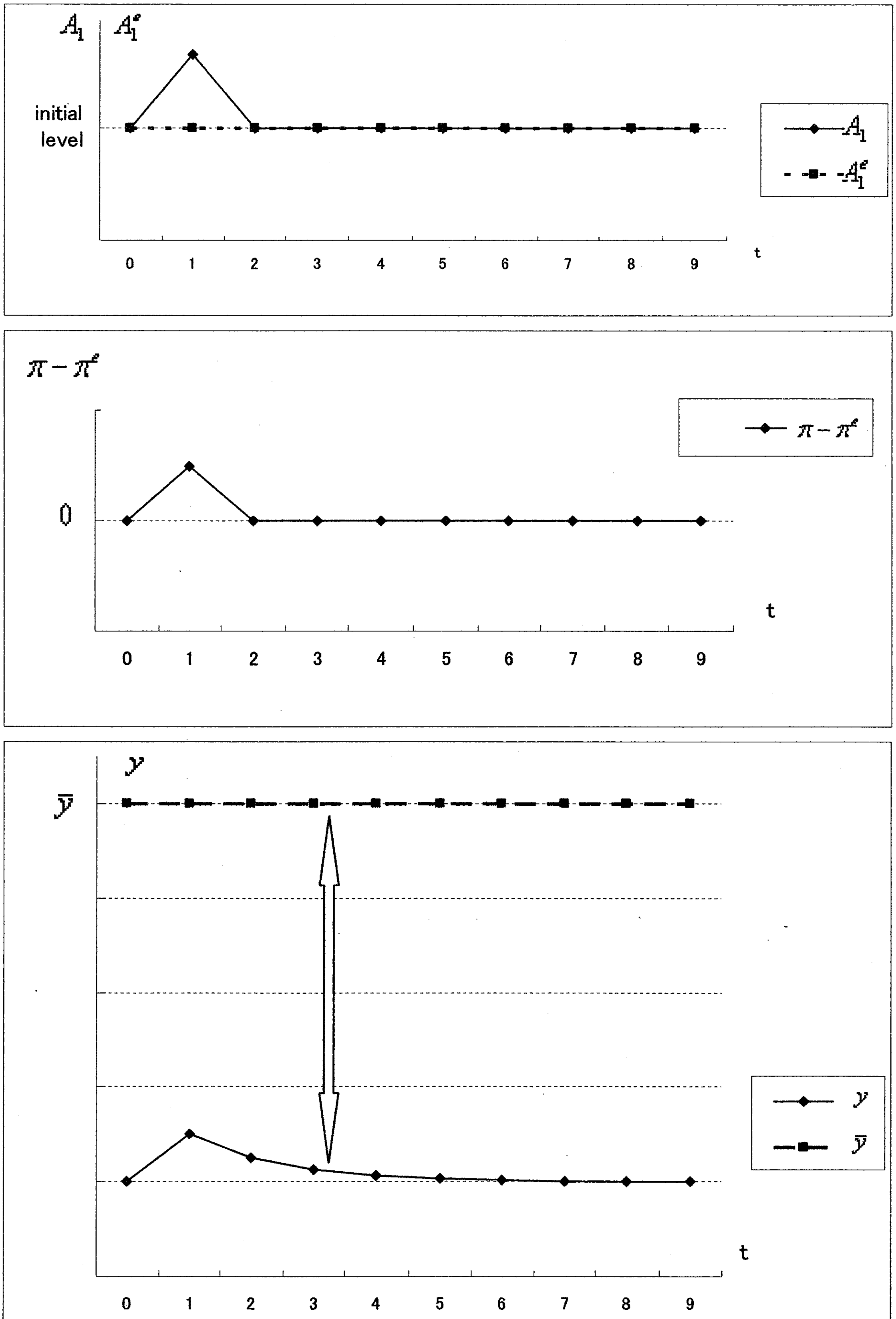


Figure IV-1 The Effects of a One-time Deviation of Inflation

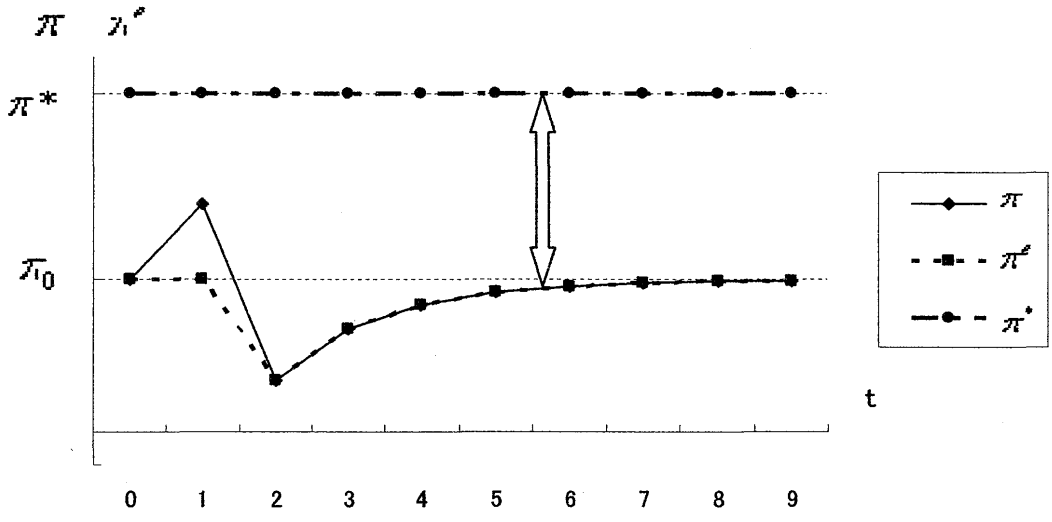
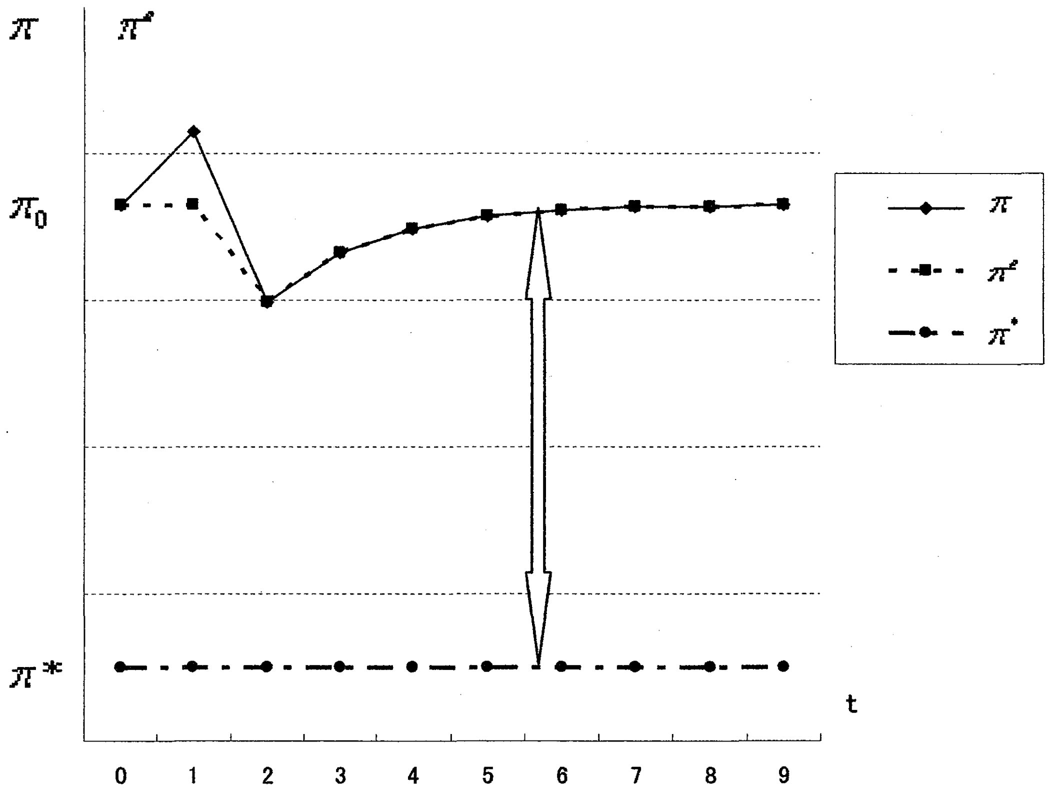


Figure IV-2 The Paths of Inflation and Expected Inflation
(A Deflationary Environment)



(An Inflationary Environment)

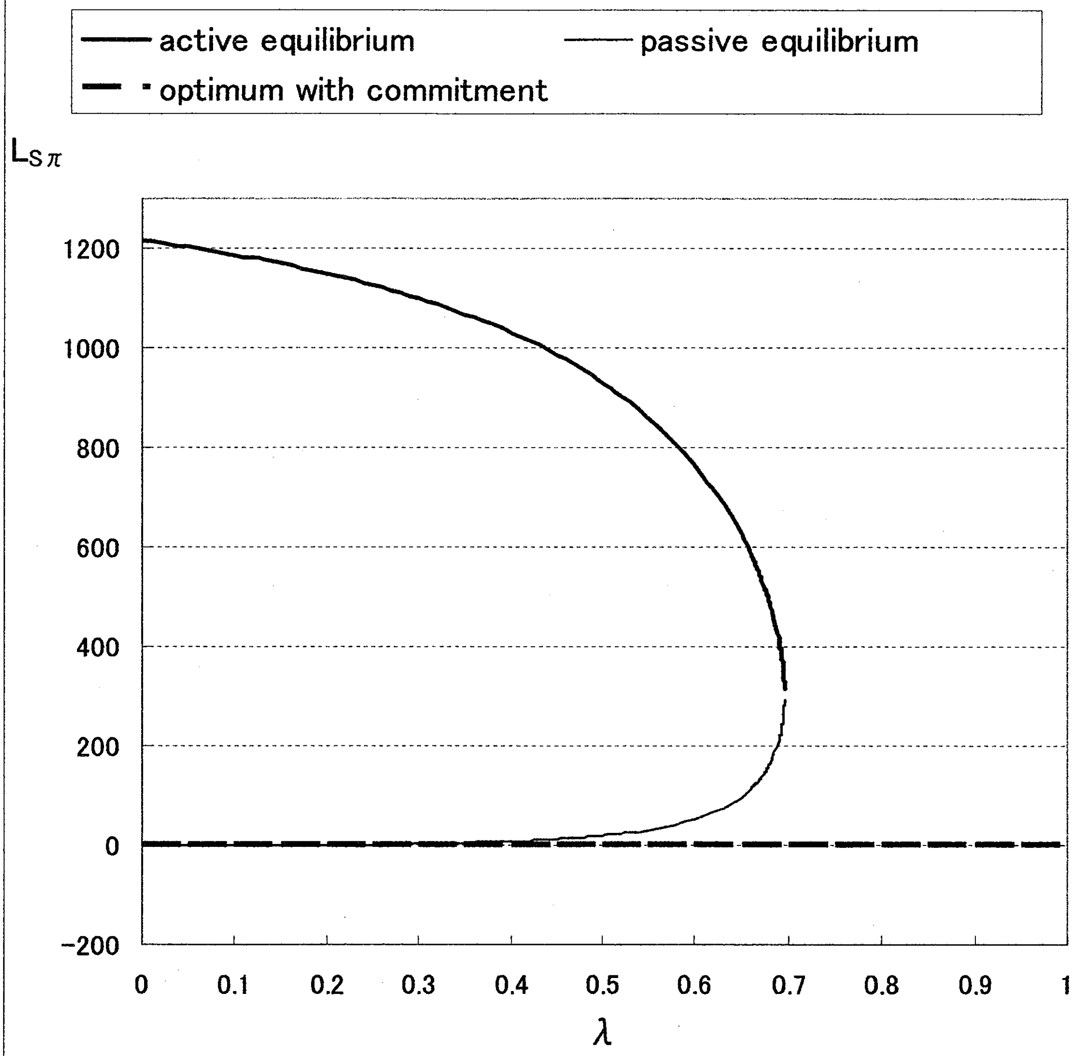


Figure V-1 Inflation Variability
(Stochastic Component)
α=0.5, ρ =0.5, β =.99

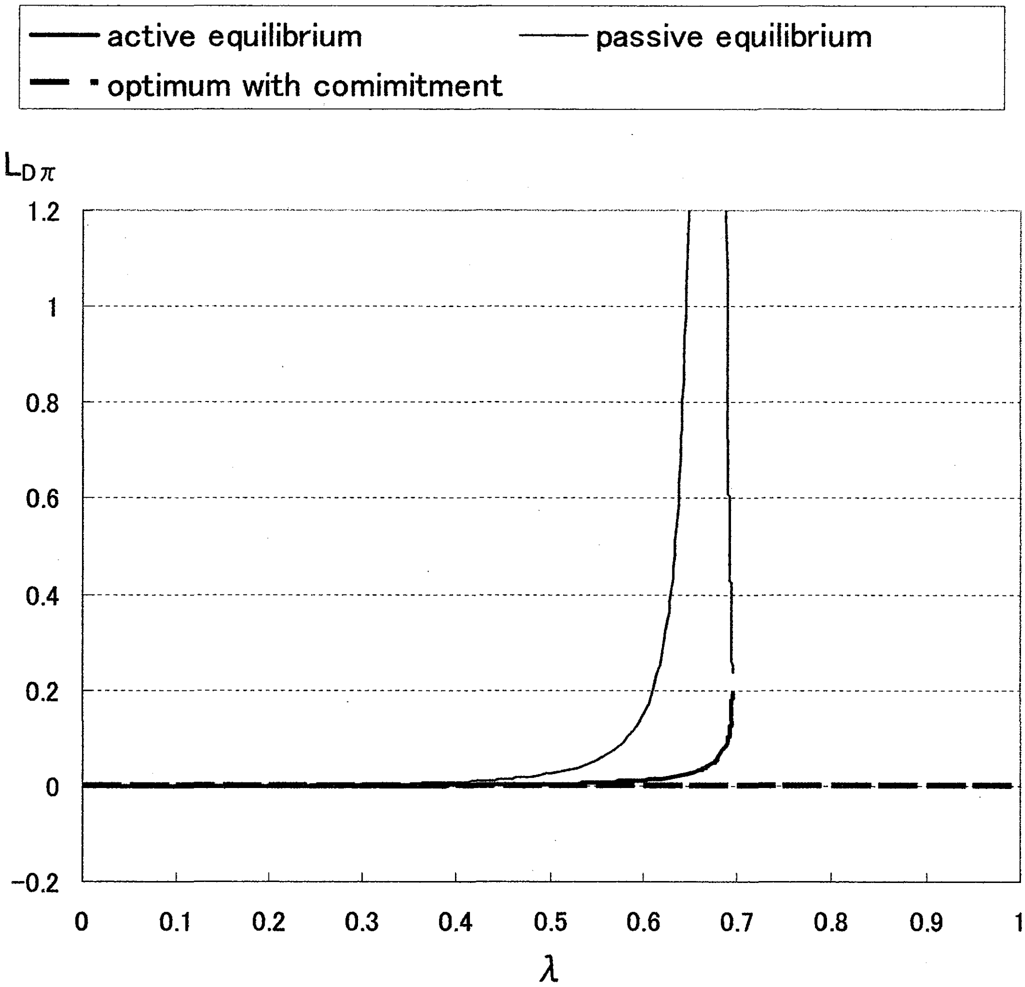


Figure V-2 Inflation Variability
(Deterministic Component)
α =0.5, ρ =0.5, β =.99

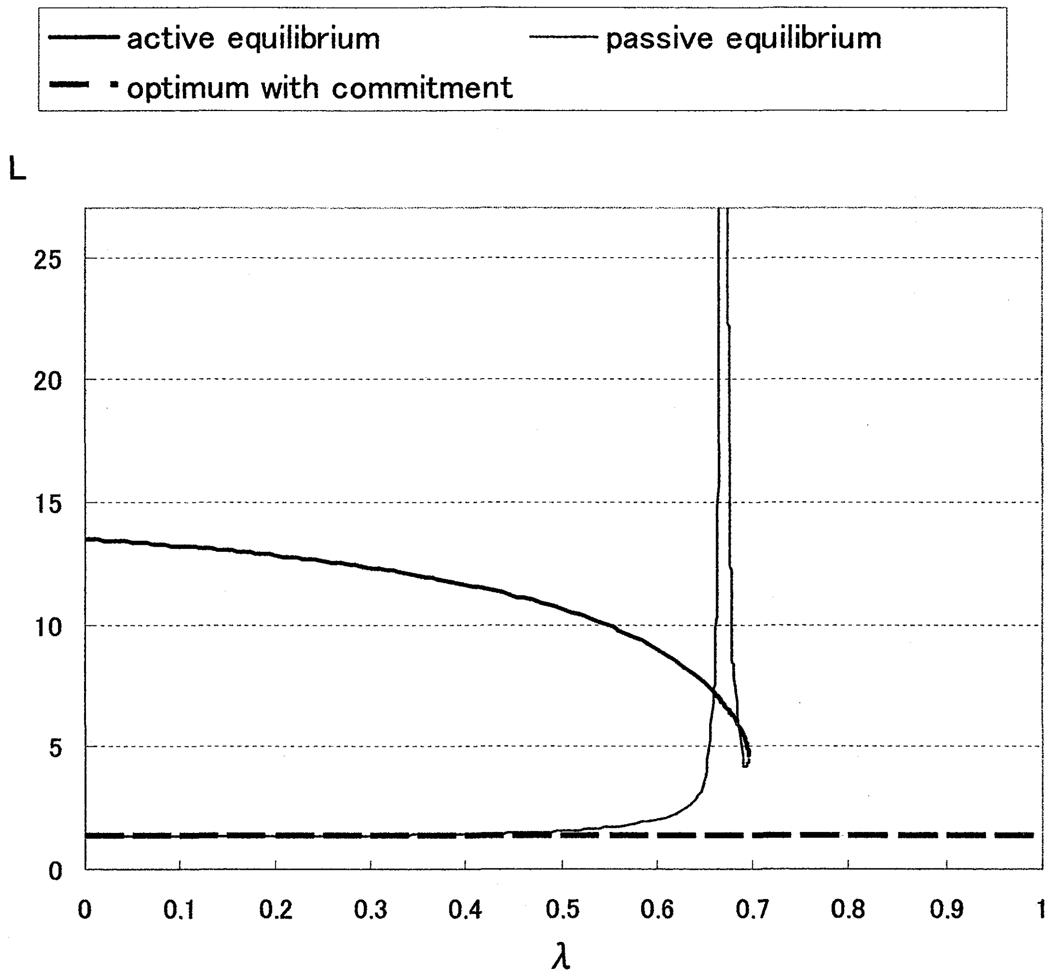


Figure V-3 Total Loss

($\sigma_{\varepsilon}{}^{2}$=0.01)

α=0.5, ρ =0.5, β =.99

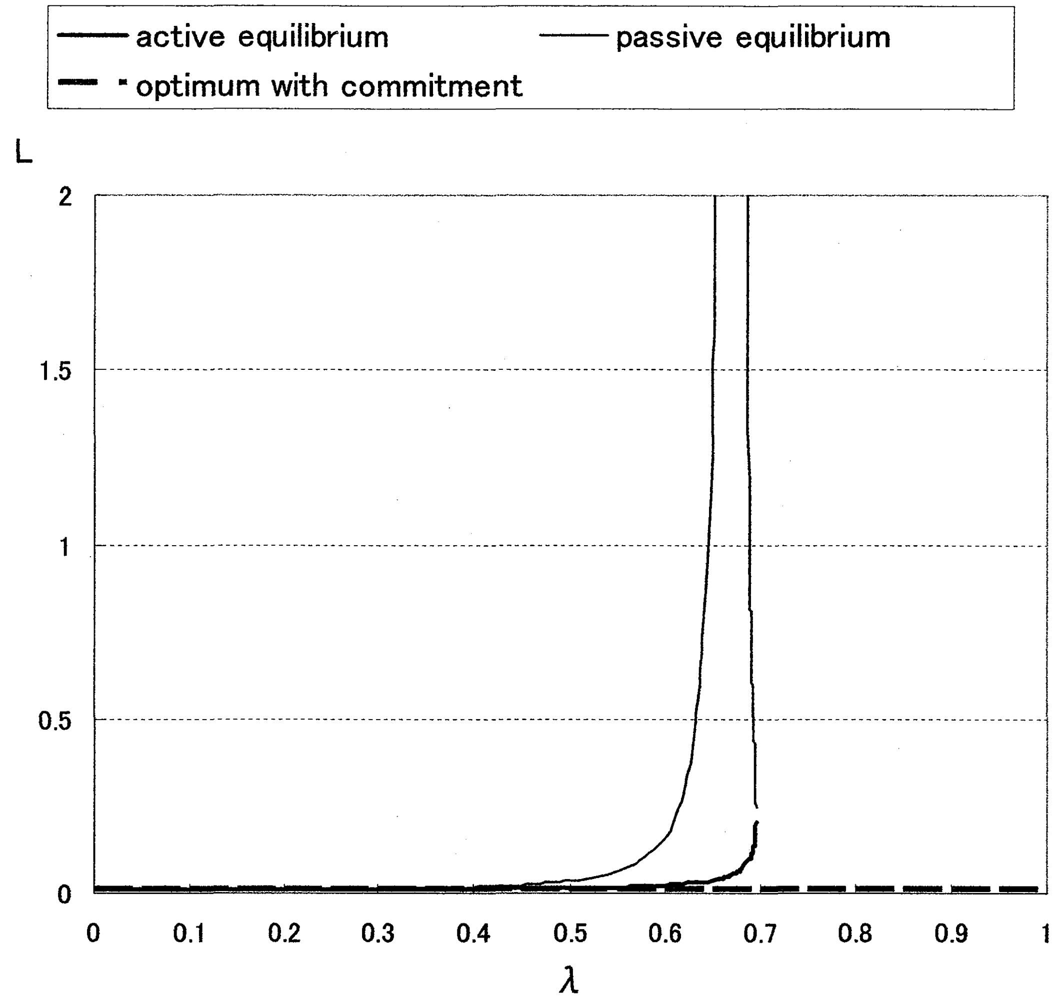


Figure V-4 Total Loss

(σ^2=0.000001)

α=0.5, ρ =0.5, β =.99