# Performance of Multiple Inspections for Product with Multiple Quality Characteristics

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#### Introduction

Two types of errors, classifying a conforming unit as nonconforming (Type I error) and classifying a nonconforming unit as conforming (Type II error), are possible in inspection. The effects of the inspection errors on the performance of inspection are studied by many researchers (Minton [1972], Collins et al. [1973], Biegel [1974], Raz and Thomas [1983], Menzefricke [1984], Maghsoodloo and Bush [1985]). However, their studies are restricted in the situation that the product has only one quality characteristic to be inspected.

Jaraiedi et al. (1987) proposed an inspection model for a product which has multiple quality characteristics which is subjected to multiple 100% inspections with Type I and Type II inspection errors. By applying the model, more practical inspection situation can be expressed. They derived the expression to compute the Average Outgoing Quality based on the model, but as shown in Appendix, their results do not correspond to the results of this paper.

In this paper, the performance of the multiple 100% inspections where each product has multiple quality characteristics to be inspected is considered. To characterize the performance of the inspection, the Average Outgoing Quality, the proportion of accepted units to lot size, the probability of accepting a nonconforming unit, and the average inspection number of quality characteristics are introduced. In addition, the optimum inspecting order of quality characteristics is proposed. From the results, these characteristics of inspections at each inspection stage can be calculated.

# Model and Inspection Plan

Suppose that a unit has J characteristics to be inspected. At the first inspection stage, an inspector examines, one by one, all characteristics of the unit in certain order. The unit is rejected as nonconforming when the inspector classifies a characteristic as nonconforming or accepted as conforming when the inspector classifies all characteristics as conforming. Accepted units are subject to inspection at the next stage and inspected in the same menner as the first stage. The inspection is continued until the desired quality level is obtained. It is assumed that probability of each characteristic is nonconforming is independent and that the inspection is independently performed at each stage. Probabilities of Type I error and Type II error are assumed to be the same for each stage.

#### Notation

N : size of the lot

J : number of quality characteristics to be inspected per unit

m : number of inspection stage

 $N_m$ : number of accepted units at stage m

 $\mathbf{e}_{1} = (\mathbf{e}_{11}, \mathbf{e}_{12}, \cdots, \mathbf{e}_{1j}, \cdots, \mathbf{e}_{1J})$ 

: probability of misclassifying conforming characteristic as nonconforming when characteristic j is inspected (Type I error)

 $\boldsymbol{e_2} \ = \ (e_{21}, \; e_{22}, \; \cdots, \; e_{2j}, \; \cdots, \; e_{2J})$ 

: probability of misclassifying nonconforming characteristic as conforming when characteristic j is inspected (Type II error)

 $\mathbf{p} = (p_1, p_2, \cdots, p_j, \cdots, p_J)$ 

: probability that characteristic j is nonconforming

AU<sub>m</sub>: proportion of accepted units to lot size at stage m

 $\mathsf{AOQ}_m$ : Average Outgoing Quality after stage m

B<sub>m</sub>: probability of accepting nonconforming unit at stage m

 $IC_m$ : average inspection number of characteristics for a unit at stage m

## Basic Properties of the Model

When a unit has one or more nonconforming characteristics, the unit is nonconforming. Chracteristic j is nonconforming with probability  $p_j$ , or conforming with probability  $1-p_j$ . Then the probability of a unit being conforming

$$p_c = \prod_{j=1}^{J} (1-p_j)$$
.

Probability of misclassifying a confaoming unit as nonconfoming

$$\alpha = 1 - \prod_{j=1}^{J} (1 - e_{1j})$$
.

Probability of classifying a unit as conforming depends on the state of the unit. To express the state of a unit and the result of inspection, Kotz and Johnson (1983) used convenient expressions, defect pattern and observed pattern.

The state of a unit is expressed by a defect pattern d.

$$\mathbf{d} = (d_1, d_2, \cdots, d_j, \cdots, d_J), \qquad d_j = 0 \text{ or } 1$$

When  $d_j = 0$ , characteristic j is conforming, and  $d_j = 1$ , then characteristic j is nonconforming. There are  $2^J$  different defect patterns. Only the unit with defect pattern  $\underline{\mathbf{d}}_0 = (0, 0, \dots, 0)$  is conforming, all others with different patterns are nonconforming.

Probability of classifying a unit with defect pattern  $\underline{\mathbf{d}}$  as conforming is given as follows.

$$\mathbf{b}_{\underline{\mathbf{d}}} = \prod_{j=1}^{J} (1 - \mathbf{e}_{1j})^{1 - \mathbf{d}_{j}} \ \mathbf{e}_{2j}^{\mathbf{d}_{j}} \tag{1}$$

 $b_{\underline{do}}$  is the prodability of correct classification for a conforming unit and equal to 1- $\alpha$ . For the other patterns,  $b_{\underline{d}}$  are the probabilities of misclassifying nonconforming units as conforming.

For a unit, the result of inspection is expressed by an odserved pattern **h**.

$$\mathbf{h} = (h_1, h_2, \dots, h_j, \dots, h_J), \quad h_j = 0 \text{ or } 1$$

When,  $h_i=0$  means that the inspector has classified the characteristic as conforming, and  $h_i=1$  means that the inspector has classified the characteristic j as nonconforming. Accepted unit has the observation pattern  $\underline{\mathbf{h}}_0=(0,\,0,\,\cdots,\,0)$ , and the unit with other observation pattern is a rejected unit.

Probability of obtaining an observation pattern  $\underline{\boldsymbol{h}}$  for a given defect pattern  $\boldsymbol{d}$ 

$$Pr(\underline{\mathbf{h}} \mid \underline{\mathbf{d}}) = \prod_{j=1}^{J} \{ e_{1j}^{(1-d_{j})h_{j}} (1-e_{1j})^{(1-d_{j})(1-h_{j})} \times e_{2j}^{d_{j}(1-h_{j})} (1-e_{2j})^{d_{j}h_{j}} \}.$$
(2)

Probability of obtaining the observation pattern  $\underline{\mathbf{h}}_0$  for a given defect pattern  $\mathbf{d}$  gives another expression for  $\mathbf{b}_d$ .

$$b_{\underline{\mathbf{d}}} = \Pr\left(\underline{\mathbf{h}}_{\mathbf{0}} \mid \underline{\mathbf{d}}\right)$$

#### Distributions Derived from the Model

The probability that the lot of size N contain  $N_0\left(\underline{\boldsymbol{d}}\right)$  units with defect pattern  $\boldsymbol{d}$ 

$$Pr\{N_{0}(\underline{\mathbf{d}}) = {N \choose N_{0}(\underline{\mathbf{d}})} g_{\underline{\mathbf{d}}}^{N_{0}(\underline{\mathbf{d}})} (1 - g_{\underline{\mathbf{d}}})^{N - N_{0}(\underline{\mathbf{d}})}$$
(3)

where

$$g_{\underline{d}} = \prod_{j=1}^{J} p_j^{d_j} (1-p_j)^{1-d_j}$$
,

$$N = \sum_{\underline{\mathbf{d}}} N_0 (\underline{\mathbf{d}}).$$

Where  $\sum_{\underline{\mathbf{d}}}$  represents  $\sum_{d_1=0}^{1}$   $\sum_{d_2=0}^{1}$   $\cdots$   $\sum_{d_j=0}^{1}$  and it means summation over all sets of  $(d_1, d_2, \dots, d_J)$ .

Expected nuber of  $N_0$  ( $\underline{\mathbf{d}}$ )

$$E \{N_0 (d)\} = N \cdot g_{d}.$$

When  $N_0$  ( $\underline{\mathbf{d}}$ ) units with defect pattern  $\underline{\mathbf{d}}$  are inspected at stage 1, the probability of accepting  $N_1$  ( $\underline{\mathbf{d}}$ ) units is

$$Pr\{N_{1}(\underline{\mathbf{d}}) \mid N_{0}(\underline{\mathbf{d}})\} = {N_{0}(\underline{\mathbf{d}}) \choose N_{1}(\underline{\mathbf{d}})} b_{\underline{\mathbf{d}}}^{N_{1}(\underline{\mathbf{d}})} (1-b_{\underline{\mathbf{d}}}).^{N_{0}(\underline{\mathbf{d}})-N_{1}(\underline{\mathbf{d}})}$$

Marginal distribution of  $N_1$  ( $\underline{\boldsymbol{d}}$ ) is as follows.

$$Pr\{N_{1}(\underline{\mathbf{d}})\} = \sum_{N_{0}(\underline{\mathbf{d}})=N_{1}(\underline{\mathbf{d}})}^{N} Pr\{N_{1}(\underline{\mathbf{d}}) \mid N_{0}(\underline{\mathbf{d}})\} Pr\{N_{0}(\underline{\mathbf{d}})\}$$

$$= {N \choose N_{1}(\underline{\mathbf{d}})} (g_{\underline{\mathbf{d}}}b_{\underline{\mathbf{d}}})^{N_{1}(\underline{\mathbf{d}})} \{1 - g_{\underline{\mathbf{d}}}b_{\underline{\mathbf{d}}}\}^{N - N(\underline{\mathbf{d}})}$$

$$(4)$$

For stage m, distribution of  $N_m$  ( $\underline{\mathbf{d}}$ ) is given by the same manner. When  $N_{m-1}$  ( $\underline{\mathbf{d}}$ ) units with defect pattern  $\underline{\mathbf{d}}$  are inspected at stage m, the probability of accepting  $N_m$  ( $\underline{\mathbf{d}}$ ) unit is

$$Pr\{N_{m}(\underline{\mathbf{d}}) \mid N_{m-1}(\underline{\mathbf{d}})\} = {N_{m-1}(\underline{\mathbf{d}}) \choose N_{m}(\underline{\mathbf{d}})} b_{\underline{\mathbf{d}}}^{N_{m}(\underline{\mathbf{d}})} (1-b_{\underline{\mathbf{d}}})^{N_{m-1}(\underline{\mathbf{d}})-N_{m}(\underline{\mathbf{d}})}$$

Marginal distribution of N<sub>m</sub> (d) is as follows.

$$Pr\{N_{m}(\underline{\mathbf{d}})\} = \sum_{N_{m-1}(\underline{\mathbf{d}})=N_{m}(\underline{\mathbf{d}})}^{N} Pr\{N_{m}(\underline{\mathbf{d}}) \mid N_{m-1}(\underline{\mathbf{d}})\} Pr\{N_{m-1}(\underline{\mathbf{d}})\}$$

$$= {N \choose N_{m}(\underline{\mathbf{d}})} (g_{\underline{\mathbf{d}}}b_{\underline{\mathbf{d}}}^{m})^{N_{m}(\underline{\mathbf{d}})} \{1 - g_{\underline{\mathbf{d}}}b_{\underline{\mathbf{d}}}^{m}\}^{N - N_{m}(\underline{\mathbf{d}})}$$
(5)

$$\begin{split} \mathbf{E}\left\{\mathbf{N}_{m}\left(\underline{\mathbf{d}}\right)\right\} &= \mathbf{N} \cdot \mathbf{g}_{\underline{\mathbf{d}}} \mathbf{b}_{\underline{\mathbf{d}}}^{m} \\ &= \mathbf{N} \prod_{i=1}^{J} \left\{ \left(1 - \mathbf{p}_{i}\right) \left(1 - \mathbf{e}_{1i}\right)^{m} \right\}^{1 - d_{i}} \left(\mathbf{p}_{i} \, \mathbf{e}_{2i}^{m}\right)^{d_{i}} \end{split}$$

(6)

where  $d_i$ 's are the elements of the defect pattern  $\underline{\mathbf{d}}$ . Especially, for  $\underline{\mathbf{d}}_0 = (0, 0, \cdots, 0)$ ,

$$E\{ N_{m} (\underline{\mathbf{d}}_{0}) \} = \prod_{j=1}^{J} (1-p_{j}) (1-e_{1j})^{m}$$

$$= N \cdot P_{c} (1-\alpha)^{m} .$$

$$(7)$$

E  $\{N_m (\underline{\mathbf{d}}_0)\}$  is the expected number of conforming units classified as conforming at stage m.

Expected total number of units classifed as conforming at atage m is

$$E(N_{m}) = \sum_{\underline{\mathbf{d}}} E\{N_{m}(\underline{\mathbf{d}})\}$$

$$= N \prod_{j=1}^{J} \{(1-p_{j}) (1-e_{1j})^{m} + p_{j} e_{2j}^{m} \}.$$
(8)

## Characteristics of Multiple Inspections

Proportion of accepted units to lot size at stage m is

$$AU_{m} = \frac{E(N_{m})}{N} = \prod_{j=1}^{J} \{ (1-p_{j}) (1-e_{1j})^{m} + p_{j} e_{2j}^{m} \}.$$
 (9)

At stage m, the expected number of units classified as conforming is given by equation (8), and the expacted number of conforming units classified as conforming is given by equation (7). Average Outgoing Quality after stage m

AOQm = 
$$1 - \frac{E\{N_{m}(\underline{\mathbf{d}}_{0})\}}{E(N_{m})}$$
  
=  $1 - \prod_{j=1}^{J} \frac{(1-p_{j})(1-e_{1j})^{m}}{(1-p_{j})(1-e_{1j})^{m}+p_{j}e_{2j}^{m}}$ . (10)

At stage m, the probability of accepting a nonconforming unit

$$B_{m} = \frac{E(N_{m}) - E\{N_{m}(\underline{\mathbf{d}}_{0})\}}{E(N_{m-1}) - E\{N_{m-1}(\underline{\mathbf{d}}_{0})\}}.$$
(11)

 $B_m$  is increasing function of m, and the upper limit of  $B_m$  (m $\rightarrow \infty$ ) is

$$\lim_{m \to \infty} B_m = e_{2j}^* \cdot \prod_{j \neq j^*} (1 - e_{1j})$$
(12)

where  $j^*$  is the number of the characteristics which has the maximum value of  $e_{2j}/(1-e_{1j})$ . Equation (12) implies that, in inspection at higher stage, the probability of accepting a nonconforming unit is close to the probability of accepting a unit with only one nonconforming characteristic  $j^*$ .

## Average Inspection Number of Characteristics

At stage m, probability of a unit has defect pattern d

$$P^{(m)}(\underline{\mathbf{d}}) = E\{N_{m-1}(\underline{\mathbf{d}})\}/E(N_{m-1})..$$
(13)

Marginal probability of a unit having an observation pattern  $\underline{\mathbf{h}}$  at stage m is given from equations (2) and (13).

$$P^{(m)}(\underline{\mathbf{h}}) = \sum_{\underline{\mathbf{d}}} P_{r} (\underline{\mathbf{h}} | \underline{\mathbf{d}}) P^{(m)}(\underline{\mathbf{d}})$$

$$= \prod_{j=1}^{J} \frac{(1-p_{j})e_{1j}^{h_{j}} (1-e_{1j})^{m-h_{j}} + p_{j} (1-e_{2j})^{h_{j}}e_{2j}^{m-h_{j}}}{(1-p_{j})(1-e_{1j})^{m-1} + p_{j}e_{2j}^{m-1}}$$
(14)

In this inspection plan, the J characteristics of a unit are inspected in certain order and the unit is rejected as nonconforming unit as soon as the inspector observes the first nonconforming characteristic. The probability that C<sup>th</sup> characteristic is the first nonconfoming characteristic to be observed correspond to the probability that the observation pattern with 0 for first C-1 and 1 for C<sup>th</sup> characteristic, and given from equation (14).

$$Q^{(m)}(C) = \frac{(1-p_c)e_{1c}(1-e_{1c})^{m-1}+p_c(1-e_{2c})e_{2c}^{m-1}}{(1-p_c)(1-e_{1c})^{m-1}+p_ce_{2c}^{m-1}} \times \prod_{j=1}^{C-1} \frac{(1-p_j)(1-e_{1j})^m+p_je_{2j}^m}{(1-p_j)(1-e_{1j})^{m-1}+p_je_{2j}^{m-1}}$$
(15)

where for  $j \ge C+1$ , the values of  $h_j$ 's are ignored.

For a unit inspected at stage m, the average inspection number of characteristics

$$IC_{m} = J \frac{E(N_{m})}{E(N_{m-1})} + \sum_{C=1}^{J} C \cdot Q^{(m)}(C).$$
(16)

IC<sub>m</sub> varies with the inspecting order of the J characteristics.

To obtain the optimum order of inspection which gives the minimum  $IC_m$ , it is necessary that

$$Q^{(m)}(1) \ge Q(2) \ge \cdots \ge Q^{(m)}(J) .$$

Therefore, at each stage, characteristics should be inspected in order of magnitude of

$$\frac{q_{j}^{(m-1)}-q_{j}^{(m)}}{q_{j}^{(m-1)}}$$

Where  $q_j^{(m)} = (1-p_j)(1-e_{1j})^m + p_j e_{2j}^m$ .

# Examples

Characteristics of multiple 100% inspections are demonstrated under various conditions of  $\mathbf{p}$ ,  $\mathbf{e}_1$  and  $\mathbf{e}_2$ . Two types of  $\mathbf{p}$  and three types of each  $\mathbf{e}_1$  and  $\mathbf{e}_2$  are assumed.

Let J=4,  $\mathbf{p}(1)=(0.01,\ 0.05,\ 0.09,\ 0.13)$ ,  $\mathbf{p}(2)=(0.071,\ 0.071,\ 0.071,\ 0.071)$ ,  $\mathbf{e}_1(1)=(0.004,\ 0.008,\ 0.012,\ 0.016)$ ,  $\mathbf{e}_1(2)=(0.01,\ 0.01,\ 0.01,\ 0.01)$ ,  $\mathbf{e}_1(3)=(0.016,\ 0.012,\ 0.008,\ 0.004)$ ,  $\mathbf{e}_2(1)=(0.01,\ 0.04,\ 0.06,\ 0.09)$ ,  $\mathbf{e}_2(2)=(0.05,\ 0.05,\ 0.05,\ 0.05)$  and  $\mathbf{e}_2(3)=(0.09,\ 0.06,\ 0.04,\ 0.01)$ . Where  $\mathbf{p}(1)$ ,  $\mathbf{e}_1(1)$ ,  $\mathbf{e}_2(1)$  are ascending type,  $\mathbf{p}(2)$ ,  $\mathbf{e}_1(2)$ ,

 $\mathbf{e_2}$  (2) are flat type and  $\mathbf{e_1}$  (3),  $\mathbf{e_2}$  (3) are descending type. All types of  $\mathbf{p}$  have common probability 1 - Pc = 0.225 of a unit being nonconforming, and the type of  $\mathbf{e_1}$  have common probability  $\alpha = 0.0394$  of misclassifying a conforming unit as nonconformig. The types of  $\mathbf{e_2}$  have common average 0.05.

For every combination of these **p**, **e**<sub>1</sub> and **e**<sub>2</sub>, AOQ<sub>m</sub> are shown in Table 1. To compare these cases, Cases 1 is considered as basic case by reson that the type of **p**, **e**<sub>1</sub> and **e**<sub>2</sub> are flat. AOQ<sub>m</sub> varies with the condition of **p**, **e**<sub>1</sub> and **e**<sub>2</sub>, however effect of the type of **e**<sub>2</sub> is not so large. For flat **e**<sub>2</sub>, type of **p** has a little effect on the AOQ, and the AOQ<sub>4</sub> are almost the same as the Case 1. For descending type of **e**<sub>2</sub> and ascending type of **p**, the AOQ are improved remarkably, and the AOQ<sub>4</sub> are reduced to about 0.84 of the Case 1. For flat **p** and ascending or descending **e**<sub>2</sub>, the AOQ<sub>4</sub> are about 3.2 times of the Case 1. Combination of ascending type of **p** and ascending type of **e**<sub>2</sub> are the worst, and the AOQ<sub>4</sub> are about 5.8 times as large as the Case 1. To improve AOQ, it is effective to reduce the error probability **e**<sub>2</sub> for the characteristic with higher proportion nonconforming.

Table 2 show the chracteristics of multiple inspections for the Case 1, 9 and 14.  $AU_m$  indicate the proportions of accepted units at stage m to lot size. There are only a little differences between cases. It might be due to the fact that these cases have common Pc and common  $\alpha$ .

Probability of accepting nonconforming unit  $B_m$  increases as m increases, and close to the limit of  $B_m$  ( $m\rightarrow\infty$ ). The limit of  $B_m$  for the Case 1, 9 and 14 are 0.0485, 0.0873 and 0.0879, respectively.

Average inspection number of characteristics IC varies with the inspection order of characteristics. The optimum inspection order of chracteristics and the minimum ICm are shown. In Case 1 with flat  $\mathbf{p}$ , flat  $\mathbf{e}_1$  and flat  $\mathbf{e}_2$ , the order of inspection does not affect on the IC. The IC is reduced to 89% of J=4 at stage 1 and to 98% at stage 2. In Case 9 with ascending  $\mathbf{p}$ , flat  $\mathbf{e}_1$  and descending  $\mathbf{e}_2$ , by changing the order of inspection in every stage, the IC is reduced to 84.5% at stage 1 and to 98% at stage 2. In Case 14 of ascending type of  $\mathbf{p}$ ,  $\mathbf{e}_1$  and  $\mathbf{e}_2$ , the inspecting order should be fixed as 4, 3, 2,

- 1. Then, the IC is reduced to 85% at stage 1 and to 97% at stage
- 2. When the optimum inspection order is used, IC shows a little differences between cases at higher stage.

## Summary

For products which have multiple quality characteristics, the performance of multiple 100% inspections with errors and the effects of errors on the performance are considered. The Average Outgoing Quality (AOQ<sub>m</sub>), the proportion of accepted units to lot size (AU<sub>m</sub>), the probability of accepting a nonconforming unit (B<sub>m</sub>) and the average inspection number of quality characteristics (IC<sub>m</sub>) are introduced to express the performance of inspections. In addition, the optimum inspection order of quality characteristics which gives the minium IC<sub>m</sub> is proposed. From the results in this paper, these characteristics of inspection at each inspection stage can be easily calculated. The effects of the Type I error and the Type II error on the inspection performance are demonstrated through numerical examples.

These results can be used to evaluate the inspection system, to improve the inspection system and to design the economical system of inspection.

# **Appendix**

Equation (11) geves the conditional probability at stage m of accepting a unit given the unit nonconforming.

From

$$B_1 = \frac{E(N_1) - E\{N_1(\underline{\mathbf{d}}_0)\}}{N(1-Pc)}$$
,

Bm is expressed as follows.

$$B_{m} = \frac{E(N_{m}) - E\{N_{m}(\underline{\mathbf{d}}_{0})\}}{N(1 - Pc)B_{1}B_{2} \cdot \cdot \cdot \cdot B_{m-1}}$$
(A. 1)

AOQ<sub>m</sub> can formally be expressed by another form by using these

 $B_m$ .

$$\begin{split} AOQ_{m} &= \frac{\{Number \ of \ accepted \ nonforming \ unit \ at \ stage \ m\}}{\{Number \ of \ accepted \ unit \ at \ stage \ m\}} \\ &= \frac{E \ (N_{m}) - E \{N_{m} \ (\underline{\boldsymbol{d}}_{0})\}}{E \{N_{m} \ (\underline{\boldsymbol{d}}_{0})\} + \{E \ (N_{m}) - E \ [N_{m} \ (\underline{\boldsymbol{d}}_{0})]\}} \\ &= \frac{(1 - P_{c}) \ B_{1}B_{2} \cdots B_{m}}{P_{c} \ (1 - \alpha)^{m} + (1 - P_{c}) \ B_{1}B_{2} \cdots B_{m}} \end{split} \tag{A. 2}$$

Jaraiedi et al. (1987) gave AOQ<sub>m</sub> as follows.

AOQ = 
$$\frac{(1-P_{c}) \beta^{m}}{P_{c} (1-\alpha)^{m} + (1-Pc) \beta^{m}}$$

They evaluated  $\beta$  as follows.

$$\beta = \sum_{k=1}^{J} P(E_k)$$
 (A.3)

where  $P(E_k)$  is the probability that there are k nonconforming characteristics present which are all misclassified and the remaining J-k conforming characteristics are correctly classified. For example,

$$P(E_1) = \sum_{i=1}^{J} p_i e_{2i} \prod_{i=1}^{J} (1-p_i) (1-e_{1j})$$

They defined  $\beta$  in their Appendix as the conditional probability of accepting a unit given the unit is nonconforming. However  $\beta$  defined by (A.3) is not the conditional probability but the probability of a unit is nonconforming and accepted at the first stage. Therefore,  $\beta$  correspond to  $(1-P_c)$   $B_1$ .

### References

Biegel, J. E. (1974). "Inspection Error and Sampling Plans". *AIIE Transactions* 6, pp.284-287.

Collins, R. D. Jr.; Case, K. E.; and Bennett, G. K.

(1973). "The Effects of Inspection Error on Single Sampling Inspection Plans." *International Journal of Production Research* 11, pp.289-298.

Jaraiedi, M.; Kochhar, D. S.; and Jaisingh, S. C. (1987).

- "Multiple Inspection to Meet Desired Outgoing Quality".

  Jorunal of Quality Technology 19, pp.46-51.
- Kotz, S.; and Jaisingh, N. L. (1983). "Some Distributions Arising from Faulty Inspection with Multitype Defectives, and an Application to Grading". Communications in Statistics, Theory and Methods, 12, pp.2809-2821.
- Maghsoodloo, S.; and Bush, B. K. (1985). "The Effects of Inspection Error on Double Sampling by Attribute". *Journal of Quality Technology* 17, pp.32-39.
- Menzefricke, U. (1984). "The Effect of Variability in Inspector Error". Journal of Quality Technology 16, pp.131-135.
- Minton, G. (1972). "Verification Error in Single Sampling Inspection Plans for Processing Survey Data". Journal of the American Statistical Association 67, pp.46-54.
- Raz, T.; and Thomas, M. U. (1983). "A Method for Sequencing Inspection Activities Subject to Errors". *IIE Transactions* 15, pp. 12-18.

**Key Word**: Average Outgoing Quality, Inspection Error, Optimum Inspecting Order of Characteristics

Table 1. Average Outgoing Quality at Stage m (aE-b =  $a \times 10^{-b}$ )

Case	p	$\mathbf{e}_1$	$\mathbf{e}_2$	m = 1	2	3	4
1	2	2	2	1.53E-2	7. 79E-4	3.94E-5	1. 99E-6
2	2	2	1	1.53E-2	1.04E-3	7.96E-5	6.46E-6
3	2	1	3	1.52E-2	1.04E-3	7.85E-5	6.32E-6
4	2	1	2	1.56E-2	7.79E-4	3.94E-5	1.99E-6
5	2	1	1	1.53E-2	1.05E-3	8.07E-5	6.59E-6
6	1	3	3	9.55E-3	4.54E-4	2.62E-5	1.69E-6
7	1	3	2	1.55E-2	7.88E-4	3.97E-5	2.00E-6
8	1	3	1	2.14E-2	1.67E-3	1.36E-4	1.14E-5
9	1	2	3	9.55E-3	4.53E-4	2.60E-5	1.67E-6
10	1	2	2	1.55E-2	7.93E-4	4.01E-5	2.02E-6
11	1	2	1	2.15E-2	1.68E-3	1.38E-4	1.17E-5
12	1	1	3	9.56E-3	4.52E-4	2.58E-5	1.65E-6
13	1	1	2	1.56E-2	7.98E-3	4.04E-5	2.05E-6
14	1	1	1	2. 15E-2	1.70E-3	1.40E-4	1. 19E-5

TaBLE 2. Characteristics of Multiple Inspections (aE-b =  $a \times 10^{-b}$ )

Case		m = 1	2	3	4
	AU,,	0. 727	0. 688	0. 660	0. 634
	$AOQ_m$	1.53E-2	7.79E-4	3.94E-5	1.99E-6
1	$B_{m}$	0. 0435	0.0482	0.0485	0.0485
	Order	In any order			•
	$\mathrm{IC}_{\mathtt{m}}$	3. 56	3. 92	3. 94	3. 94
	AUm	0.722	0. 687	0.660	0. 634
	AOQ <sub>m</sub>	9.55E-3	4. 53E-4	2. 60E-5	1.67E-6
9	Bm	0. 0270	0.0452	0.0550	0.0616
	Order	4, 3, 2, 1	3, 2, 4, 1	2, 3, 1, 4	2, 1, 3, 4
	IC <sub>m</sub>	3. 38	3. 92	3. 94	3. 94
	AUm	0. 731	0. 688	0.660	0. 634
	AOQ <sub>m</sub>	2. 15E-2	1.70E-3	1.40E-4	1.19E-5
14	Bm	0.0617	0.0742	0.0790	0. 0820
	Order	4, 3, 2, 1	4, 3, 2, 1	4, 3, 2, 1	4, 3, 2, 1
	IC <sub>m</sub>	3. 39	3. 87	3. 92	3. 92