

Formulae for Spot Price and Forward Curve in Wholesale Electricity Markets

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Abstract

This paper examines the factors that determine the equilibrium spot and forward prices in a wholesale electricity market. In order to investigate this issue, we focus on two characteristics of electricity: the rule of non-storability and the balancing rule, which is simultaneous equal quantities of demand and supply. The Bessembinder and Lemmon model (the BL model) certainly considers these characteristics in order for determining the equilibrium spot and forward prices. Next, we generalize the BL model in order to analyze the electricity market consisting of power producing firms with different cost functions in a multi-period model, and we draw the equilibrium spot and forward electricity price formulae. Furthermore, we reveal that in certain cases, the division of a firm leads to a higher average spot price and causes several more spikes in the spot price process.

KEYWORDS: electricity market, spot price, forward price, spike, division

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1 Introduction

During the last few decades, the problem regarding the frameworks of electricity trading and designing of the trading markets has been discussed. From the research or practicality viewpoint, there are two important problems in the wholesale electricity markets – the manner of determining spot and forward prices and the manner of using electricity forward curves (term structure) in power risk management. The aim of this paper is to investigate the former problem, that is, “how equilibrium spot and forward prices would be determined in wholesale electricity markets.” This problem will be discussed by our proposed model.

We focus on two characteristics of electricity. The first is that electricity cannot be stored, which we term as the non-storability rule. The second is the balancing rule that the supply of power must be equal to the demand at any given time. It appears as though these two characteristics would largely affect the trading strategies and the spot and forward prices in wholesale electricity markets. In fact, the main problem associated with pricing electricity derivatives based on the hedging strategy does not capture the unique features of electricity. In derivative pricing that is based on no-arbitrage, we assume that every investor can take short and long positions in the underlying asset. However, in contrast to other financial market instruments, e.g., shares, bonds, foreign currencies, and commodities, we find that it is impossible to take these positions in the electricity market due to non-storability. This demonstrates that it is not reasonable to apply the widely-used option pricing theory to electricity derivatives.

The Bessembinder and Lemmon model (2002), hereafter referred to as the BL model, takes an equilibrium approach to derive the spot and forward price formulae in a wholesale electricity market. In the BL model, two unique features of electricity are considered. Using the BL model’s equilibrium pricing formula, it is possible to explain the spikes in the electricity spot price processes. Spikes have been observed in some electricity markets, and their descriptions in an economic model constitutes one of the important problems. Note that in the BL model, power generating and retailing firms are assumed to be homogeneous. On the other hand, Green and Newbery (1992), Borenstein and Bushnell (1999), and so on suggest that spot prices are primarily affected by market power with different product abilities. In order to discuss further problems a model with

different power producers is required.

In order to derive more practical equilibrium price formulae in a wholesale electricity market, we generalize the BL model in the following manner. In the BL model, the cost functions and risk preferences for all power generating firms are assumed to be homogeneous, similar to power retailing firms. However, in our model these assumptions are relaxed. That is, we consider that cost functions and risk preferences are different for power producers and retailers, and we also consider various distributions of power generating firms. Additionally, our model is a multi-period model. Based on these considerations particularly, it would be possible to discuss the problems related to market structure with regard to different power generating firms. Considering the existence of different product abilities among power generating firms, in other words, considering the manner in which the various distributions of power generating firms with different product abilities affect equilibrium spot prices, would have meaningful political implications.

This paper is structured as follows: We present our model in Section 2, we construct a framework for spot pricing in Section 3, and we present a forward curve formula in Section 4. These discussions include many cost functions and distributions of generating firms in order that our model can involve various market structures. In Section 5, we specify the cost functions and distributions of power generating firms and present numerical examples of spot price distributions using our equilibrium pricing formulae. Next, we examine the manner in which the equilibrium spot price would be varied by different market structures that represent different cost functions of power generating firms. We also consider the political implications arising from these examinations. In Section 6, we present the conclusion and the topic for further research.

2 The model

In this section, we describe the assumptions and notations used throughout this paper.

On a wholesale spot market, power generating firms trade electricity among themselves and with retailers who distribute power to their customers. The trades and the distribution take place at discrete times, i.e., our model is a multiperiod model. At each time, producers and retailers

can buy or sell forward contracts for their risk management. Therefore, all power producers select their output and forward portfolios on each trading date. Due to the non-storability rule, these selections do not influence each other. On the other hand, retailers purchase the difference between the realized retail demand and their previous forward positions, and select forward portfolios for future demand. For simplicity, we assume that the firms have different power production cost functions and the power distribution companies have different retail prices. However, these do not change throughout the trading period.

The set of nonnegative real numbers is denoted by \mathbf{R}_+ and the set of nonnegative members of the extended line $\overline{\mathbf{R}}$ is denoted by $\overline{\mathbf{R}}_+$. For any topological space Ψ , $\mathcal{B}(\Psi)$ denotes the Borel σ -field on Ψ .

Let (Ω, \mathcal{F}, P) be a probability space. $M, N \in \mathbf{N}$ are interpreted as the number of power generating firms and that of retailers, respectively.

Let μ and ν denote two probability measures on the product measurable space $(\mathbf{R}^n, \mathcal{B}(\mathbf{R}^n)) \times (\mathbf{R}_+, \mathcal{B}(\mathbf{R}_+))$. We define two probability measures μ_1 and ν_1 on $(\mathbf{R}^n, \mathcal{B}(\mathbf{R}^n))$ as follows:

$$\begin{aligned}\mu_1(A) &= \mu(A \times \mathbf{R}_+) \quad \text{for } A \in \mathcal{B}(\mathbf{R}^n), \\ \nu_1(A) &= \nu(A \times \mathbf{R}_+) \quad \text{for } A \in \mathcal{B}(\mathbf{R}^n).\end{aligned}$$

Furthermore, we define two probability measures μ_2 and ν_2 on $(\mathbf{R}_+, \mathcal{B}(\mathbf{R}_+))$ in a similar manner:

$$\begin{aligned}\mu_2(A) &= \mu(\mathbf{R}^n \times A) \quad \text{for } A \in \mathcal{B}(\mathbf{R}_+), \\ \nu_2(A) &= \nu(\mathbf{R}^n \times A) \quad \text{for } A \in \mathcal{B}(\mathbf{R}_+).\end{aligned}$$

We shall explain the implications of these probability spaces. In our model, every $(\xi_1, \xi_2) \in \mathbf{R}^n \times \mathbf{R}_+$ is interpreted as a power generating firm with supply ability ξ_1 and willingness to take risk ¹ ξ_2 . Hereafter, the firm is referred to as the power producer (ξ_1, ξ_2) . The measure μ represents the distribution of M power producers classified according to both supply ability and risk aversion. μ_1 is the marginal distribution of the supply ability, and μ_2 is related to risk aversion.

A similar explanation holds for each $(\zeta_1, \zeta_2) \in \mathbf{R}^n \times \mathbf{R}_+$. This element is interpreted as a power-distribution company with retail ability ζ_1 and willingness to take risk ζ_2 , and we refer to it as the retailer (ζ_1, ζ_2) . Here,

¹The greater the valued ξ_2 , the more risk-averse is the firm. See Section 4 for details.

the retail ability represents the retailer's bargaining power in price negotiation with his/her customers and his/her selling techniques which include advertising, salesperson commissions, and so on. It affects the electricity demand to the retailer. We shall designate ν as the distribution of N retailers with respect to both retail ability and risk aversion. Measures ν_1 and ν_2 are the marginal distributions of retail ability and risk aversion, respectively.

Let $MC : \mathbf{R}_+ \times \mathbf{R}^n \rightarrow \overline{\mathbf{R}}_+$ be a measurable function with the following three conditions:

$$\begin{aligned} MC(z, \xi_1) &\text{ is continuous and strictly increasing with respect to } z, \\ MC(0, \xi_1) &= 0, \\ \lim_{z \rightarrow \infty} MC(z, \xi_1) &= +\infty, \end{aligned}$$

for each $\xi_1 \in \mathbf{R}^n$. We define $C : \mathbf{R}_+ \times \mathbf{R}^n \rightarrow \overline{\mathbf{R}}_+$ as

$$C(z, \xi_1) = \begin{cases} \eta(\xi_1) + \int_0^z MC(s, \xi_1) ds & \text{for } 0 \leq z < \rho(\xi_1) \\ \eta(\xi_1) + \lim_{y \rightarrow \rho(\xi_1)-0} \int_0^y MC(s, \xi_1) ds & \text{for } z = \rho(\xi_1) \\ +\infty & \text{for } z > \rho(\xi_1) \end{cases} \quad (1)$$

for any $\xi_1 \in \mathbf{R}^n$, where $\eta : \mathbf{R}^n \rightarrow \mathbf{R}_+$ is a measurable function, and

$$\rho(\xi_1) = \sup\{z \in \mathbf{R}_+ \mid MC(z, \xi_1) < +\infty\}.$$

For all $\xi_1 \in \mathbf{R}^n$, we use $C(z, \xi_1)$ as the cost function of each power generating firm with supply ability ξ_1 and $MC(z, \xi_1)$ as the marginal cost function.

Let a measurable function $p : \mathbf{R}^n \rightarrow \mathbf{R}_+$ have a finite integral with respect to ν_1 . p is referred to as the retail price curve; in other words, every firm with retail ability ζ_1 sells electric power at $p(\zeta_1)$ per 1 unit to his/her consumers.

Let $Y : \mathbf{N} \times \Omega \times \mathbf{R}^n \times \mathbf{R}_+ \rightarrow \mathbf{R}_+$ be measurable, and

$$X(t, \omega) := \int_{\mathbf{R}^n} Y(t, \omega, \zeta_1, p(\zeta_1)) \nu_1(d\zeta_1) < \infty, \quad (2)$$

for $\forall t \in \mathbf{N}$, and P -a.s. $\omega \in \Omega$. For each $t \in \mathbf{N}$ and $\zeta_1 \in \mathbf{R}^n$, $Y(t, \zeta_1, p(\zeta_1)) := Y(t, \cdot, \zeta_1, p(\zeta_1))$ signifies the amount of electricity that

the customers of the retailer ξ_1 will demand at time t . The value of $Y(t, \zeta_1, p(\zeta_1))$ depends on the trend of all the customers' demand to the retailer ζ_1 , which is stochastic. Though natural, both the retail ability ζ_1 and the price $p(\zeta_1)$ affect the fluctuation. Then, our model contains a case that a difference in retail ability induces a difference in electricity demand among retailers even if retail prices are equal. We notice that $N \cdot X(t)$ denotes the total system's retail electricity demand at time t in our model.

Let $\mathcal{F}_0 := \{\emptyset, \Omega\}$, and $\mathcal{F}_t := \sigma(Y(1, \cdot, \cdot), Y(2, \cdot, \cdot), \dots, Y(t, \cdot, \cdot))$ for $t \in \mathbf{N}$. \mathcal{F}_t is interpreted as the information available to all market participants to make investment decisions at time $t = 0, 1, 2, \dots$

We fix $T \in \mathbf{N}$ throughout this paper. In our model, for every $t = 0, 1, 2, \dots$, electricity forward contracts with maturities or delivery dates in $t + 1, t + 2, \dots$, and $t + T$ are tradable at time t .

Let $\varphi, \psi : (\{0\} \cup \mathbf{N}) \times \mathbf{N} \times \Omega \times \mathbf{R}^n \times \mathbf{R}_+ \rightarrow \mathbf{R}$ be measurable functions that satisfy the following three conditions. The first condition is that for $\forall t = 0, 1, 2, \dots, \forall t' = t + 1, t + 2, \dots, t + T, \forall (\xi_1, \xi_2), (\zeta_1, \zeta_2) \in \mathbf{R}^n \times \mathbf{R}_+$, $\varphi(t, t', \xi_1, \xi_2)$ and $\psi(t, t', \zeta_1, \zeta_2)$ are \mathcal{F}_t -measurable functions. Second, for $\forall t = 0, 1, 2, \dots, \forall t' = t + 1, t + 2, \dots, t + T$,

$$M \int_{\mathbf{R}^n \times \mathbf{R}_+} \varphi(t, t', \xi_1, \xi_2) \mu(d(\xi_1, \xi_2)) + N \int_{\mathbf{R}^n \times \mathbf{R}_+} \psi(t, t', \zeta_1, \zeta_2) \nu(d(\zeta_1, \zeta_2)) = 0 \quad (3)$$

$P - \text{a.s.}$

Third, for $\forall t = 0, 1, 2, \dots, \forall t' = t + T + 1, t + T + 2, \dots$, $\forall (\xi_1, \xi_2), (\zeta_1, \zeta_2) \in \mathbf{R}^n \times \mathbf{R}_+$,

$$\varphi(t, t', \xi_1, \xi_2) = \psi(t, t', \zeta_1, \zeta_2) = 0.$$

We state the interpretation of φ and ψ . For each $t = 0, 1, 2, \dots$ and each $t' = t + 1, t + 2, \dots, t + T$, at time t , the producer (ξ_1, ξ_2) agrees to deliver an amount $\varphi(t, t', \xi_1, \xi_2)$ at future time t' at the fixed price. $\psi(t, t', \zeta_1, \zeta_2)$ denotes the quantity sold (purchased if negative) forward with the maturity t' by the retailer (ζ_1, ζ_2) at time t . Equation (3) is referred to as the market clearing condition, which is interpreted as equating the sum of the forward positions across producers and retailers to zero. Therefore, for every (φ, ψ) that satisfies the above three conditions, $\varphi(\cdot, \cdot, \xi_1, \xi_2)$ and $\psi(\cdot, \cdot, \zeta_1, \zeta_2)$ describe, respectively a forward strategy of the power producer (ξ_1, ξ_2) and of the retailer (ζ_1, ζ_2) .

Finally, we shall designate a positive number r as the instantaneous interest rate.

3 The spot price process in an equilibrium

For any $t \in \mathbf{N}$, any $u, v_0, v_1, \dots, v_{t-1} \geq 0$, and any $(\xi_1, \xi_2) \in \mathbf{R}^n \times \mathbf{R}_+$, we solve the following maximization problem:

$$\begin{aligned} \text{maximize} \quad & ux + \sum_{j=0}^{t-1} v_j \varphi(j, t, \xi_1, \xi_2) - C\left(x + \sum_{j=0}^{t-1} \varphi(j, t, \xi_1, \xi_2), \xi_1\right) \\ \text{subject to} \quad & x + \sum_{j=0}^{t-1} \varphi(j, t, \xi_1, \xi_2) \in [0, \rho(\xi_1)]. \end{aligned} \tag{4}$$

We shall explain the meaning of the above optimization problem (4). In this problem,

$\{v_0, v_1, \dots, v_{t-1}\}$ is the historical path of a forward price with maturity time t , which all market participants observe at the present time t . The power producer (ξ_1, ξ_2) has previously taken a position $\varphi(j, t, \xi_1, \xi_2)$ in forward with maturity t since time j . Given a present spot price u , the producer decides electricity production x in order to maximize his/her profit at time t . If the producer (ξ_1, ξ_2) generates more electricity than the maximizer, the excess would only decrease the profit because of his/her inability to stock it for future demand. A similar explanation holds for electricity generation lesser than the maximizer since short positions are impossible in electricity spot markets. Therefore, problem (4) is an expression of the non-storability rule.

It is easy to solve the above problem and the solution is

$$x^*(t, u, \xi_1, \xi_2) := g(u, \xi_1) - \sum_{j=0}^{t-1} \varphi(j, t, \xi_1, \xi_2), \tag{5}$$

where $g(\cdot, \xi_1)$ is the inverse function of $\text{MC}(\cdot, \xi_1)$ whose domain is restricted to $[0, \rho(\xi_1))$. (5) denotes each producer's supply under the spot price u . Then we define

$$\begin{aligned} G(u) &:= \int_{\mathbf{R}^n \times \mathbf{R}_+} x^*(t, u, \xi_1, \xi_2) + \sum_{j=0}^{t-1} \varphi(j, t, \xi_1, \xi_2) \mu(d(\xi_1, \xi_2)) \\ &= \int_{\mathbf{R}^n} g(u, \xi_1) \mu_1(d\xi_1) \end{aligned} \tag{6}$$

for $u \geq 0$, and $M \cdot G(u)$ represents total electricity supply when the spot price is equal to u .

Property 1 G has the following three properties:

1. $G(0) = 0$.
2. G is an increasing function.
3. If there exists $u' > 0$ such that $G(u') < +\infty$, then G is continuous on $[0, u']$.

Definition 1 Let $\gamma = \sup\{u \geq 0 \mid G(u) < +\infty\}$. The structure of the wholesale electricity market (μ_1, ν_1, MC, Y) is referred to as spot tradable if

$$P \left(\bigcap_{t=1}^{+\infty} \{N \cdot X(t) < M \cdot G(\gamma-)\} \right) = 1.$$

Furthermore, when this condition holds, we shall say that

$$U(t) := \frac{G^{(-1)} \left(\frac{N \cdot X(t)}{M} \right) + G_s^{(-1)} \left(\frac{N \cdot X(t)}{M} \right)}{2} \quad (7)$$

is the equilibrium spot price at time $t = 1, 2, 3, \dots$, and $U = (U(1), U(2), U(3), \dots)$ is the equilibrium spot price process, where

$$\begin{aligned} G(\gamma-) &= \lim_{u \rightarrow \gamma-0} G(u), \\ G^{(-1)}(z) &= \inf\{u \geq 0 \mid G(u) \geq z\}, \\ G_s^{(-1)}(z) &= \sup\{u \geq 0 \mid G(u) \leq z\}. \end{aligned}$$

It is clear that for any $t = 1, 2, 3, \dots$

$$N \cdot X(t) = M \cdot G(U(t)). \quad (8)$$

Definition 1 contains a few noteworthy factors. Equation (8) shows that the equilibrium spot price equates the total supply and demand at each time.

Some cases, in which the structure of the whole sale electricity market is not spot tradable, have an important implication. If

$$P \left(\bigcup_{t=1}^{+\infty} \{M \cdot \sup_{u \in \mathbf{R}_+} G(u) < N \cdot X(t)\} \right) > 0, \quad (9)$$

then any higher prices might fail to match supply with demand because the total electricity supply is limited.

Some literatures frequently cite the causes of spot price spikes or non-tradable situations to be the execution of market power, such as quantity control and increase in the mark-up rate. However, from an examination of the results of some data analysis or the mark-up rate in the electricity industry, it appears as though existence of such market power executions is not always investigated.

In our model, spot prices are determined by the competitive equilibrium, where the market marginal cost curve intersects the demand curve under the “non storability rule” and the “balancing rule.” Therefore, we do not assume the existence of market power, which is represented by increasing the mark-up or quantity control under Cournot competition. In our setting, spikes of spot prices or non-tradable situations might occur even in a competitive market. Therefore, we suggest that it is not reasonable to explain the cause of spikes “only” by the execution of market power. These results include important implications from the policy viewpoint.

4 The forward price curve in an equilibrium

In this section, we shall derive an equilibrium forward price curve. Hereafter, we assume that the structure of a wholesale electricity market (μ_1, ν_1, MC, Y) is spot tradable, all conditional expectations and covariances are finite, and interchange of the order of integration in multiple integrals is allowed.

We begin by considering the forward portfolio selection problems faced by both producers and retailers. Then, based on the results, we will derive the forward price curve.

Let any $t = 0, 1, 2, \dots$ be fixed. Given any $\{v_1, v_2, \dots, v_T\}$ and $\{z_{j,t+k}\}_{j=0,1,\dots,t-1,k=1,2,\dots,T}$ in \mathbf{R}_+ . Here, $\{v_1, v_2, \dots, v_T\}$ has an interpretation of a forward price curve realized at time t , that is, a forward price with delivery time $t+k$ is v_k at time t . $z_{j,t+k}$ is interpreted as a price of the forward contract which was initiated at time j and will be settled at time $t+k$.

For $\mu - \text{a.e. } (\xi_1, \xi_2) \in \mathbf{R}^n \times \mathbf{R}_+$, we first solve the following:

$$\begin{aligned} \text{maximize} \quad & E \left({}^t \mathbf{a}(t, \xi_1, \xi_2) D \mathbf{1} - {}^t (\mathbf{U}(t) - \mathbf{v}) D \mathbf{y} \mid \mathcal{F}_t \right) \\ & - \frac{1}{2} \xi_2 \text{Var} \left({}^t \mathbf{a}(t, \xi_1, \xi_2) D \mathbf{1} - {}^t (\mathbf{U}(t) - \mathbf{v}) D \mathbf{y} \mid \mathcal{F}_t \right) \quad (10) \\ \text{subject to} \quad & \mathbf{y} \in \mathbf{R}^T, \end{aligned}$$

where the transpose of a matrix A is denoted by ${}^t A$,

$$\begin{aligned} a(t, k, \xi_1, \xi_2) &= U(t+k)g(U(t+k), \xi_1) - C(g(U(t+k), \xi_1), \xi_1) \\ &\quad - \sum_{j=0}^{t-1} (U(t+k) - z_{j,t+k}) \phi(j, t+k, \xi_1, \xi_2) \quad \text{for } k \in \mathbf{N}, \\ \mathbf{a}(t, \xi_1, \xi_2) &= \begin{pmatrix} a(t, 1, \xi_1, \xi_2) \\ a(t, 2, \xi_1, \xi_2) \\ \vdots \\ a(t, T, \xi_1, \xi_2) \end{pmatrix}, \quad \mathbf{U}(t) = \begin{pmatrix} U(t+1) \\ U(t+2) \\ \vdots \\ U(t+T) \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_T \end{pmatrix}, \end{aligned}$$

D is a $T \times T$ diagonal matrix whose diagonal entries are $e^{-r}, e^{-2r}, \dots, e^{-rT}$, and $\mathbf{1}$ is a T -dimensional vector with all elements equal to 1. Then, for $\nu - \text{a.e. } (\zeta_1, \zeta_2) \in \mathbf{R}^n \times \mathbf{R}_+$, we will find the maximizer of the following problem :

$$\begin{aligned} \text{maximize} \quad & E \left({}^t \mathbf{b}(t, \zeta_1, \zeta_2) D \mathbf{1} - {}^t (\mathbf{U}(t) - \mathbf{v}) D \mathbf{y} \mid \mathcal{F}_t \right) \\ & - \frac{1}{2} \xi_2 \text{Var} \left({}^t \mathbf{b}(t, \zeta_1, \zeta_2) D \mathbf{1} - {}^t (\mathbf{U}(t) - \mathbf{v}) D \mathbf{y} \mid \mathcal{F}_t \right) \quad (11) \\ \text{subject to} \quad & \mathbf{y} \in \mathbf{R}^T, \end{aligned}$$

where

$$\begin{aligned} b(t, k, \zeta_1, \zeta_2) &= \{p(\zeta_1) - U(t+k)\} Y(t+k, \zeta_1, p) \\ &\quad - \sum_{j=0}^{t-1} (U(t+k) - z_{j,t+k}) \psi(j, t+k, \zeta_1, \zeta_2) \quad \text{for } k \in \mathbf{N}, \\ \mathbf{b}(t, \zeta_1, \zeta_2) &= \begin{pmatrix} b(t, 1, \zeta_1, \zeta_2) \\ b(t, 2, \zeta_1, \zeta_2) \\ \vdots \\ b(t, T, \zeta_1, \zeta_2) \end{pmatrix}. \end{aligned}$$

Next, we require to interpret the maximizing problem (10). The power producer (ξ_1, ξ_2) is aware of the information \mathcal{F}_t , which contains the historical and present electricity forward price curves. He/she may utilize the present forward price curve $\mathbf{v} = {}^t(v_1, v_2, \dots, v_T)$ in order to control the uncertainty generated by the power spot price process. He/she can only manage his/her future profit flow that will be received until time $t + T$, namely, $\mathbf{a}(t, \xi_1, \xi_2)$. We notice that ${}^t\mathbf{a}(t, \xi_1, \xi_2)D\mathbf{1}$ represents the sum of the discounted values of those future profits. Then, he/she decides to take forward short positions \mathbf{y} (purchased if the element is negative) for maximizing the objective function. A similar interpretation holds for (11).

Both the optimization problems (10) and (11) are easily solved. The solutions of (10) and (11) are

$$-\frac{1}{\xi_2}D^{-1}\Sigma(t)^{-1}\{E(\mathbf{U}(t)|\mathcal{F}_t) - \mathbf{v}\} + D^{-1}\Sigma(t)^{-1}\Gamma(t, \xi_1, \xi_2)D\mathbf{1} \quad (12)$$

and

$$-\frac{1}{\zeta_2}D^{-1}\Sigma(t)^{-1}\{E(\mathbf{U}(t)|\mathcal{F}_t) - \mathbf{v}\} + D^{-1}\Sigma(t)^{-1}\Lambda(t, \zeta_1, \zeta_2)D\mathbf{1}, \quad (13)$$

respectively, where $\Sigma(t)$, $\Gamma(t, \xi_1, \xi_2)$, and $\Lambda(t, \zeta_1, \zeta_2)$ are square matrices of order T , the entry in position (k, l) of $\Sigma(t)$ is

$$\text{Cov}(U(t+k), U(t+l) | \mathcal{F}_t),$$

the entry in position (k, l) of $\Gamma(t, \xi_1, \xi_2)$ is

$$\text{Cov}(a(t, k, \xi_1, \xi_2), U(t+l) | \mathcal{F}_t),$$

and the entry in position (k, l) of $\Lambda(t, \zeta_1, \zeta_2)$ is

$$\text{Cov}(b(t, k, \zeta_1, \zeta_2), U(t+l) | \mathcal{F}_t).$$

Given the forward price curve \mathbf{v} , (12) and (13) show the optimal forward strategy of the producer (ξ_1, ξ_2) and the retailer (ζ_1, ζ_2) , respectively. For each $k = 1, 2, \dots, T$, we denote the k th entry of the vector (12) by $\varphi^*(t, t+k, \xi_1, \xi_2)$ and similarly the k th entry of the vector (13) by $\psi^*(t, t+k, \zeta_1, \zeta_2)$.

Furthermore, we will find \mathbf{v} such that

$$M \int_{\mathbf{R}^n \times \mathbf{R}_+} \varphi^*(t, t+k, \xi_1, \xi_2) \mu(d(\xi_1, \xi_2)) + N \int_{\mathbf{R}^n \times \mathbf{R}_+} \psi^*(t, t+k, \zeta_1, \zeta_2) \nu(d(\zeta_1, \zeta_2)) = 0 \quad (14)$$

for all $k = 1, 2, \dots, T$. We will briefly describe the significance of equation (14). In the forward market, every forward price can be determined by equating the sum of the positions across all the market participants to zero. Therefore, the vector \mathbf{v} satisfying (14) represents the equilibrium forward price curve in the situation wherein all producers and retailers select the optimum forward portfolios.

Using (6), (8), and interchanging the order of the integration, we obtain the solution

$$\mathbf{F}(t) := E(\mathbf{U}(t) | \mathcal{F}_t) - \frac{\Theta(t)D\mathbf{1}}{M \int_{\mathbf{R}_+} \frac{1}{\xi_2} \mu_2(d\xi_2) + N \int_{\mathbf{R}_+} \frac{1}{\zeta_2} \nu_2(d\zeta_2)}, \quad (15)$$

where $\Theta(t)$ is a square matrix whose (k, l) -th entry is

$$\begin{aligned} & \text{Cov} \left(N \int_{\mathbf{R}^n} p(\zeta_1) Y(t+k, \zeta_1, p) \nu_1(d\zeta_1) \right. \\ & \quad \left. - M \int_{\mathbf{R}^n} C(g(U(t+k), \xi_1), \xi_1) \mu_1(d\xi_1), U(t+l) \middle| \mathcal{F}_t \right) \end{aligned}$$

for $k, l = 1, 2, \dots, T$. Thus, we shall say that $\mathbf{F}(t)$ is the equilibrium forward curve at time t or the forward price formula. There exists an important aspect regarding the forward price formula.

See

$$N \int_{\mathbf{R}^n} p(\zeta_1) Y(t+k, \zeta_1, p) \nu_1(d\zeta_1) - M \int_{\mathbf{R}^n} C(g(U(t+k), \xi_1), \xi_1) \mu_1(d\xi_1), \quad (16)$$

where the first term indicates the total income of all retailers by selling electricity $N \cdot X(t+k)$ and the second term shows the total cost incurred by all the producers in generating electricity $M \cdot G(U(t+k))$. Then, (16) is interpreted as the profit of the electricity industry at time $t+k$. (15) describes the difference between the forward prices and the expected spot prices in terms of the covariances of the total profits and the equilibrium spot prices, and the degree of risk aversion. The forward price formula is consistent with recent researches. Leong (1997) indicates that there exist qualitative differences between a forward price and a forecast spot price, and the former contains the current market reality and market participants' willingness to transact. Pirrong and Jermakyan (1999) observe that the price of a forward with one-day maturity is a biased predictor

of the realized spot price over the period from 1997-98. They note that a systematic difference or the market price of risk causes the difference. The second term of (15) explicitly describes the structure of the differences which these studies refer to. It is true that Bessembinder and Lemmon (2002) modeled it; however, the equilibrium forward curve (15) is more general.

5 Examining distributions of spot prices with numerical examples

Until the previous section, we derive the equilibrium spot price and forward curve. In this section, we specify the distributions of power generating firms and retailing firms and their cost functions. Next, using numerical examples ², we investigate the manner in which the distributions of spot prices are affected by the difference among the abilities of power generating firms, which is expressed by different cost functions.

We provide the structure of a wholesale electricity market (μ_1, ν_1, MC, Y) in the following manner. We set $0 \leq \xi_1 \leq \xi_2$, $\alpha > 0$ and $M_1, M_2, N \in \mathbf{N}$. We define two probability measures μ and ν on $(\mathbf{R}, \mathcal{B}(\mathbf{R})) \times (\mathbf{R}_+, \mathcal{B}(\mathbf{R}_+))$ as

$$\begin{aligned} \mu(\{(\xi_1, \alpha)\}) &= \frac{M_1}{M}, & \mu(\{(\xi_2, \alpha)\}) &= \frac{M_2}{M}, \\ \nu(\{(1, \alpha)\}) &= \nu(\{(2, \alpha)\}) = \dots = \nu(\{(N, \alpha)\}) &= \frac{1}{N}, \end{aligned}$$

where $M = M_1 + M_2$. We shall explain the interpretation on μ . There are two groups of power producers. One group comprises of M_1 firms with supply ability ξ_1 and the other group comprises of M_2 firms with supply ability ξ_2 . Note that the case wherein $\xi_1 = \xi_2$ involves the BL model. Then, for $j = 1, 2$

$$\eta(\xi_j) = \theta, \tag{17}$$

$$MC(z, \xi_j) = cz^{1+\xi_j} \quad \text{for } z \geq 0, \tag{18}$$

²Our model contains such a great number of cost functions and producers' distributions that it is applicable to analysis of various electricity market structures. This section presents only a few examples.

where c and θ are positive constants. $\xi_1 \leq \xi_2$ represents the difference between the two groups. A firm with marginal cost (18) is termed as a power generating firm ξ_j .

Next, we describe the settings of retail firms. We set $N = 20$, $p(1) = p(2) = \dots = p(N) = p_0$ (a positive constant) and assume that $\{Y_j(t) := Y(t, j, p_0) \mid j = 1, 2, \dots, N, t = 1, 2, 3, \dots\}$ is a set of independent and $N(5, 4)$ -distributed random variables. Let

$$X(t) := \frac{1}{N}Y_1(t) + \frac{1}{N}Y_2(t) + \dots + \frac{1}{N}Y_N(t),$$

which is related to the amount of retailer firms' demand ³.

In Figure 1, we draw the graph of $MC(z, \xi) = cz^{1+\xi}$ for each $\xi = 0, 0.3, 0.5, 1, 3$, where $c = 0.605$ ⁴. Each $\xi = 0, 0.3, 0.5, 1, 3$ corresponds to thick, broken, one-point chain, dotted, and thin curved line, respectively. The lower the valued ξ , the more (cost) efficient is the firm.

We consider five numerical examples based on these settings.

The first case is $(M_1, M_2, \xi_1, \xi_2) = (1, 19, 0, 3)$ in which there are 20 power generating firms in the market and one firm is efficient with supply ability $\xi_1 = 0$, and the remaining 19 firms have ability $\xi_2 = 3$.

The second case is $(M_1, M_2, \xi_1, \xi_2) = (2, 19, 0.2, 3)$. In this case, one efficient firm with supply ability $\xi_1 = 0$ (as in the first case) divides into 2 firms with $\xi_1 = 0.2$, and the remaining 19 firms have equiability as in the first case.

The density functions of spot prices in the first and second cases are drawn in Figure 2. On comparing the graph before dividing the firms (solid line) to the one after the division (broken line), it is observed that the center of the distribution of the spot prices in the latter case is positioned on the left-hand side. Therefore, the average price level in the latter case is roughly lower than that in the former. In other words, dividing

³By the additional assumption, $U(1), U(2), \dots$ are i.i.d. random variables.

⁴0.605 is an approximate value to the solution of an equation

$$20 \cdot 5 = 20 \cdot \left\{ \frac{1}{20} \left(\frac{30}{c} \right)^{\frac{1}{1+0}} + \frac{19}{20} \left(\frac{30}{c} \right)^{\frac{1}{1+3}} \right\}$$

for c . In the first case, which we will explain below, we select c such that the spot price is 30, i.e., $U(t) = 30$ in the event that the electricity demand is equal to $N \cdot E(X(t))$. Bessembinder and Lemmon (2002) contains a similar numerical example.

firms has a positive effect in decreasing the level of spot price.

The third case is $(M_1, M_2, \xi_1, \xi_2) = (2, 19, 0.3, 3)$, which implies that one efficient firm with supply ability $\xi_1 = 0$ is divided into 2 firms. These 2 firms are less efficient than those in the second case. In Figure 3, we draw the density functions of the spot prices in the first and third cases.

In Figure 3, the solid line represents the density function before the division and the broken line represents one after the division. In Figure 3, the opposite result is obtained as compared to Figure 2. In the case wherein division leads to degree of efficiency loss in the cost functions (increase in ξ), in other words, when firms' product abilities decrease below a certain level due to the division and can not maintain their efficiency, the distribution of prices would be shifted to the right, and becomes the right-hand side fat tail. Such a result roughly implies increasing average prices, greater fluctuation in prices, and a tendency for spikes to occur frequently. This numerical example suggests that the effects of dividing firms in order to promote competition might possibly offset the negative effect of decreased efficiency.

We also compare $(M_1, M_2, \xi_1, \xi_2) = (4, 19, 1, 3)$ with $(M_1, M_2, \xi_1, \xi_2) = (8, 19, 3, 3)$. In the former case, one power generating firm with efficient ability $\xi_1 = 0$ is divided into 4 firms with supply ability $\xi_1 = 1$ and the remaining 19 firms have supply ability $\xi_2 = 3$. In the latter case, one firm with ability $\xi_1 = 0$ is divided into 8 firms with $\xi_1 = 3$ and the remaining 19 firms also have $\xi_2 = 3$ hence, all the 27 firms in the market have the same ability $\xi_1 = \xi_2 = 3$. These graphs in Figure 4 represent the density functions of the spot prices in each case. The former case corresponds to one-point chain line and the latter case corresponds to the broken line.

Figure 4 indicates the following: In the case of dividing firms and rendering their cost functions homogenous by reducing their product abilities, the center of the distribution of spot price would be shifted to the right, i.e., the average level of spot price would increase. Further, the distribution of spot price would be fat tail and spot prices would be unstable in such case.

With respect to market power, it is occasionally argued that dividing firms might make markets competitive. In such arguments, it is intended that some regulatory authority can control the distributions of spot prices by controlling the distribution of firms. This is because, for the purpose of enhancing economic welfare, the regulatory authority might adapt some

policies such as reducing the average price level or preventing spikes. However, in the electricity markets, a regulatory authority cannot intervene in order to control prices by trading, such as in shares or foreign currencies. Therefore, the principal means of achieving the purpose of a regulatory authority with regard to electricity spot prices would be to control the distribution of firms by using antitrust policies. However, it is at least suggested by our numerical examples that dividing firms, which is intended to promote competition, would not always lead to decreased average spot prices and increased economic welfare. Rather, it is possible that dividing firms might cause spot price fluctuations and occurrences of price spikes more frequently. There exist certain arguments that it is required to maintain the quality of electricity and stable provisions in the electricity market, these results have important implications to argue these issues.

Figure1 Marginal Cost Curves

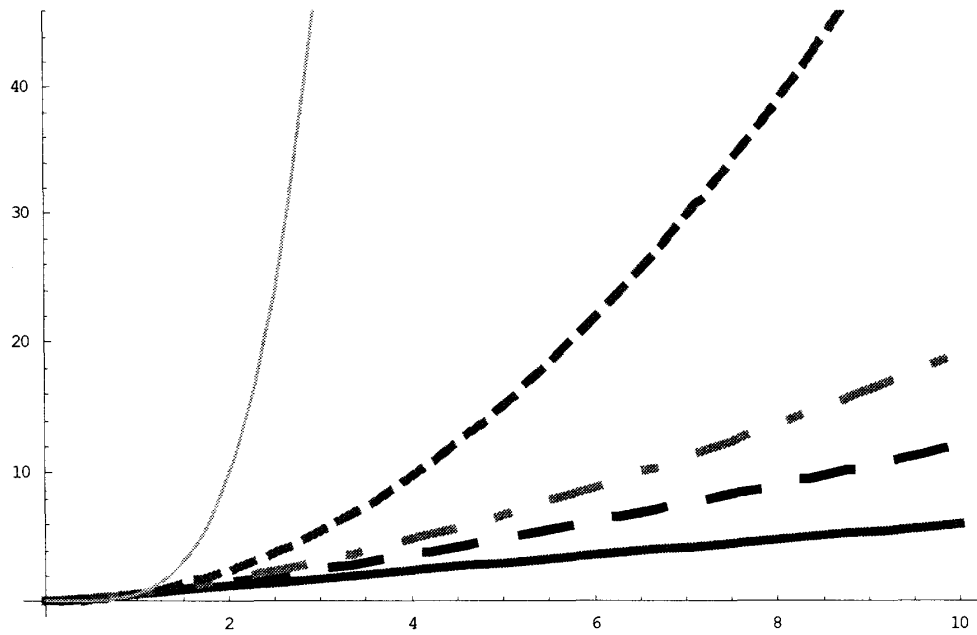


Figure 2 comparison of the case $(M_1, M_2, \xi_1, \xi_2) = (1, 19, 0, 3)$ with the case $(2, 19, 0.2, 3)$

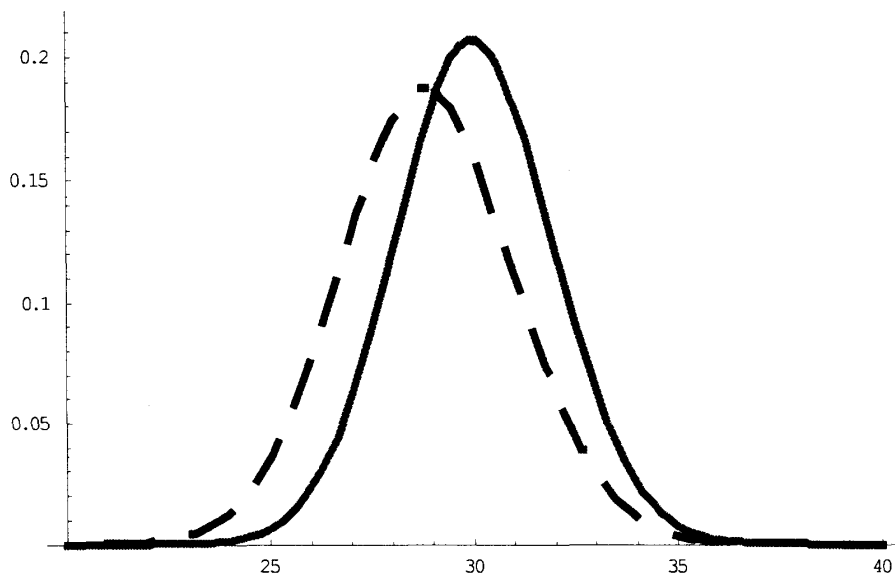


Figure 3 Comparison of the case $(M_1, M_2, \xi_1, \xi_2) = (1, 19, 0, 3)$ with the case $(2, 19, 0, 3)$

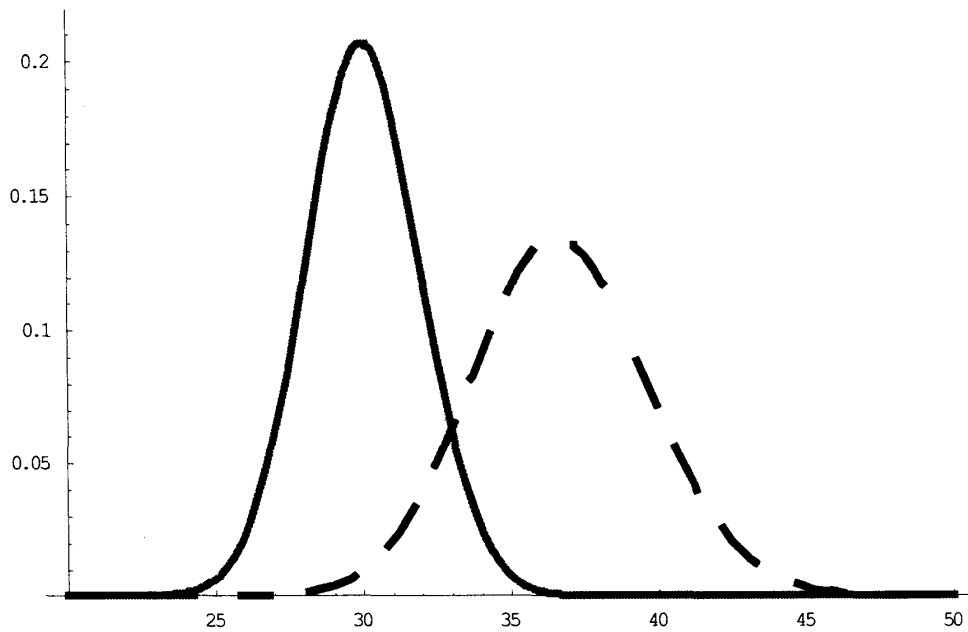
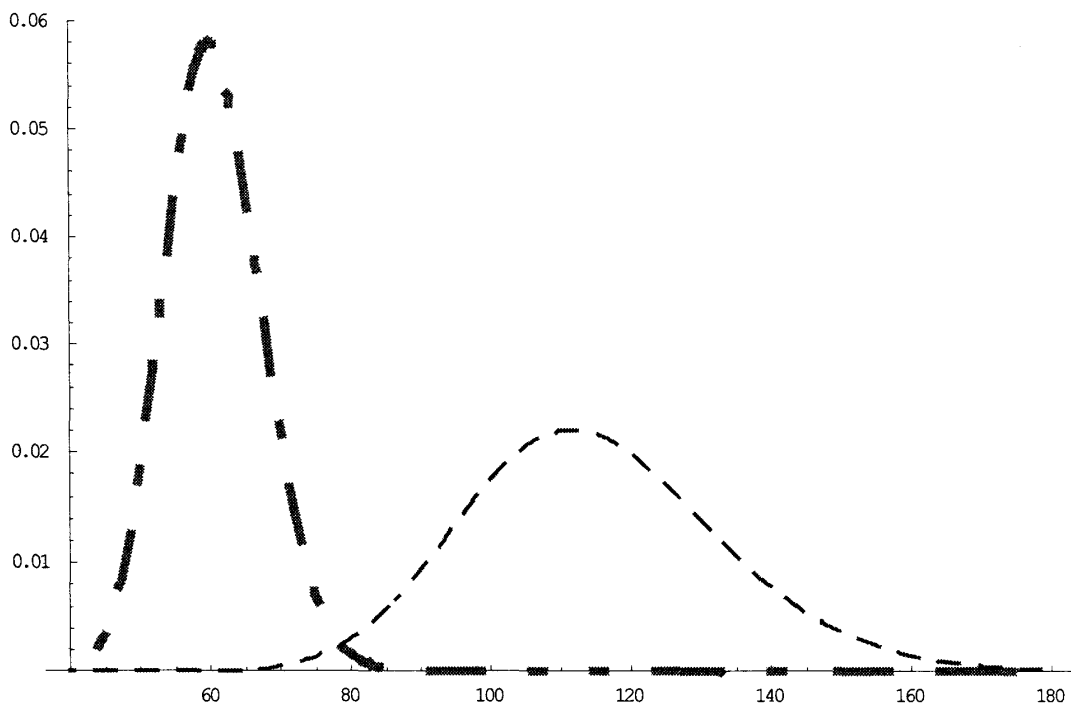


Figure 4 Comparison of the case $(M_1, M_2, \xi_1, \xi_2) = (4, 19, 1, 3)$ with $(8, 19, 3, 3)$



6 Conclusion and future research

Bessembinder and Lemmon (2002) derived the equilibrium spot and forward price formulae in a wholesale electricity market model that captures the characteristics of electricity, i.e., the “non-storability” and the “balancing rule.” We build a generalized BL model that represents the difference in ability among power producers and retailers, and we derive the equilibrium spot price and forward curve formulae. Next, using numerical examples from the viewpoint of political economics, we analyze that changes in the power producer’s distribution affect the spot price distribution.

We obtain the policy implications that have two main contributions. First, by our equilibrium pricing formulae, we suggest the possibilities that spikes of spot prices or non spot tradable situations exist. Note that in our model, these phenomena could occur even in a competitive market equilibrium. In fact, we do not directly assume strategic behaviors on the part of firms and market power execution to explain spikes. Second, dividing power generating firms would not always decrease the levels of spot prices. In some cases, dividing the firms could make the right-hand side tail of the spot price distribution relatively thick. That is, the average levels of spot prices would tend to be high and spikes of spot prices would tend to occur more frequently. These results would provide useful points to discuss electricity trading markets or the market structure of power producers.

The following problems would be dealt with in our future research:

First, one of the main problems left to be addressed is to examine relation of our equilibrium pricing formulae to market power. We have discussed under the conditions that the equilibrium spot prices are determined in order to equate the amount of power generated to the amount of realized demand. From the viewpoint of economics, our model is classified as the competitive demand-supply equilibrium model. One of our contributions is to show that spikes of spot prices could occur in such competitive environments. However it would be required to demonstrate the relation of our model to other markets such as an oligopolistic market, where the market power is considerable, and also compare the equilibrium spot and forward prices. Next, we will extend and generalize our model by incorporating the major oligopoly models such as Cournot model or

the supply function equilibrium model.

In order to discuss market power, we have to pay attention to transmission. In order to address problems regarding transmission, the characteristics of networks should be considered. Therefore, we do not address them. However, these problems still persist.

Second, in our settings, retail prices are treated as exogenous. With regard to the demand side of electricity, we do not provide any specifications regarding retailer or consumer behavior. Neither consumers' nor retailers' responses to the spot prices have been formulated. The relationship between the wholesale market and the retail market is required to be observed. We will address the problem regarding the manner in which the retail price to consumer is determined under the Cournot competition.

Third, the practical applicability of our model to electricity markets is required to be considered. Further, it is necessary to conduct data analysis and examine how our equilibrium spot and forward prices can be used to explain the prices in a practical electricity market. This data analysis will help not only to confirm the applicability of our model but also to consider the manner in which these prices can be used in firms' risk management.

Other problems remain unsolved. However, it suggests that there is room for developing our model.

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