Construction of Fuzzy Hierarchical Model

——Application to Resource Allocation Problem——

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Abstract
Attempting to translate mathematically the inarticulate elements of the human ego, experience, and volition, in the process of decision making, is prone to be difficult. As such, there has been a tendency to avoid focusing on this element in the ill-defined problem. This paper will attempt to fill the void by formulating a fuzzy hierarchical model. The model will incorporate a fuzzy inference mechanism and incentive control when examining the function of exchange of information between upper- and lower-level decision-makers. An algorithm which directly reflects the decision-maker's experience and knowledge in problem solving will be formulated. The validity of the algorithm will be inspected by application to a tangible problem, that is, the resource allocation problem.

Keyword: ill-defined problem, hierarchical model, inference mechanism, incentive, decision-maker

1. Introduction

For the most part, optimization of system has focused on well-defined problems, namely; goals, conditional limitations, problem-solving techniques, policy or evaluation methods. In practice, however, these same areas are not so clearly defined for business managers - in other words to management they actually represent ill-defined problems. This paper will broaden the attention on the hierarchical optimization of resource allocation to include other general issues such as decision-making, i.e. an ill-defined problem. Attempting to translate mathematically the inarticulate elements of the human ego, experience, and volition, in the process of decision making, is prone to be difficult. As such, there has been a tendency to avoid focusing on this element in the equation.

In general, several criteria such as Laplace criterion, maximin profit criterion, Hurwicz criterion and minimax lose criterion have been proposed to solve the ill-defined problems mentioned above. However, the results yielded by making use of these criteria will not be entirely satisfactory because their criteria won't meet the ill-defined problem with uncertainty. By looking at some of the problems encountered by management, this paper will attempt to fill the void by formulating a fuzzy hierarchical model. The model

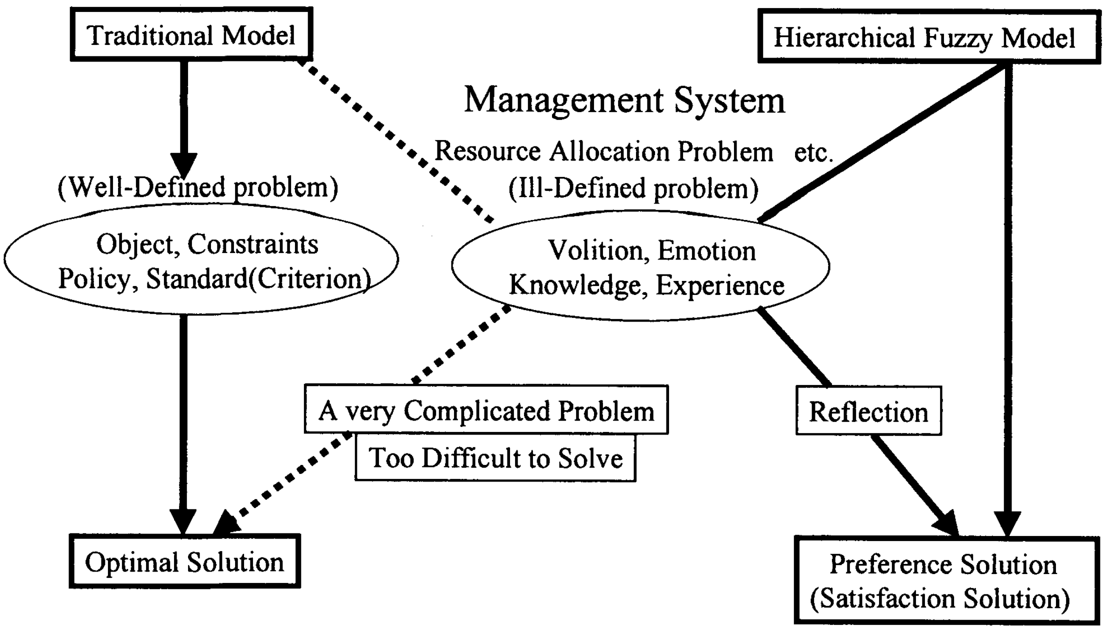

Fig.1 Optimization Problem of Hierarchical System

will incorporate a fuzzy inference mechanism and incentive control when examining the function of exchange of information between upper- and lower-level decision-makers. An algorithm which directly reflects the decision-maker's experience, volition, emotion and knowledge in problem solving will be formulated. The validity of the algorithm will be inspected by applying the algorithm to a tangible problem.

2. Preliminaries for Construction of the Fuzzy Hierarchical Model

In this section, some properties of fuzzy theory will be described as preliminaries for the construction of the fuzzy hierarchical model.

2.1 Fuzzy Inference Mechanism

The fuzzy inference rule [2] is expressed as follows:

「IF x is A_1 and y is B_1 Then z is C_1 else IF x is A_2 and y is B_2 Then z is C_2 else

… … … IF x is A_n and y is B_n Then z is C_n else x is A' and y is B' , z is C'」

where each of A_1 ,…, A_n , A' is a subset of universe of discourse U , and B_1 ,…, B_n , B' fuzzy subset of universe of discourse V ;C_1 ,…,C_n, C' subset of universe of discourse W.

「IF x is A_1 and y is B_1」 is expressed by membership function

$\mu_{A_i \cap B_i}(u,v) = \mu_{A_i}(u) \wedge \mu_{B_i}(v)$. Then $A_i \cap B_i \to C_i$ is expressed as follows:

$$\mu_{A_i \cap B_i \to C_i}(u,v,w) = [\mu_{A_i}(u) \wedge \mu_{B_i}(v)] \to \mu_{C_i}(w).$$

'else' is recognized as 'or($\cup$)' on the basis of Mandani's notation.

$$C' = (A' \cap B') o [(A_1 \cap B_1 \rightarrow C_1) \cup \ldots \cup (A_n \cap B_n \rightarrow C_n)]$$
$$= (A' \cap B') o [(A_1 \cap B_1 \rightarrow C_1) \cup \ldots \cup (A' \cap B')\ o\ (A_n \cap B_n \rightarrow C_n)]$$
$$= C_1' \cup \ldots \cup C_n'$$

C_i is expressed by making use of max-min composition as follows:

$$\mu_{Ci}'(w) = \mu_{(A' \cap B')\ o\ (An \cap Bn \rightarrow Cn)}(w) = max\{ \mu_{(A' \cap B')}(u,v) \wedge \mu_{Ai \cap Bi \rightarrow Ci}(u,v,w) \}$$
$$= \mu_{A' o (Ai \rightarrow Ci)}(w) \wedge \mu_{B' o (Bi \rightarrow Ci)}(w).$$

$\mu_{Ci}'(w)$ is also rewritten as follows:

$$\mu_{Ci}'(w) = max\{ \mu_{A'}(u) \wedge \mu_{Ai}(u) \} \wedge max\{ \mu_{B'}(v) \wedge \mu_{Bi}(v) \}\ \mu_{Ci}(w)$$

Therefore $C' = C_1 \cup C_2 \cup \ldots \cup C_n$

$$\mu_{Ci}'(w) = \mu_{C1}'(w) \vee \mu_{C2}'(w) \vee \ldots \vee \mu_{Cn}'(w)$$

2.2 Fuzzy Number

The fuzzy number of triangular type [3,4] is often used in the fields of decision-making. In this section the fuzzy number of triangular type A is described as follows:

$$A = (A_L, A_C, A_R)_L$$

and it is also can be abbreviated by

$$A = (A_L, A_C, A_R)$$

where A_C : Center of fuzzy number A , A_R : Right boundary value of fuzzy number A , A_L : Left boundary value of fuzzy number A.

The membership function of fuzzy number is shown by $\mu_A(x)$:

$$\mu_A(x) = L(x) = \begin{cases} L_1(x) & : x \in [A_C, A_R] \\ L_2(x) & : x \in [A_L, A_C] \end{cases}$$

where $L(x)$ is called type function with the following properties :

$L(A_C) = 1$, $L(A_R) = L(A_L) = 0$, $L_1(x)$ is a linear function that strictly decreases on the right interval [A_C , A_R], and $L_2(x)$ is also linear but strictly increases on the left interval [A_L , A_C]. Figure 2 shows the shape of a fuzzy number of triangular type.

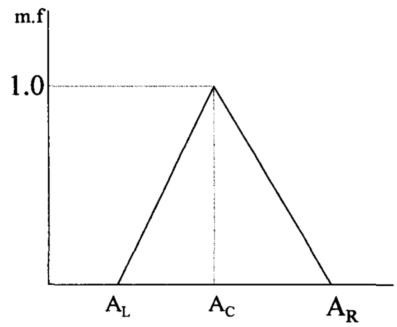

Fig.2 Fuzzy Number of Triangular Type

The type function in the above case can be expressed by the linear equations as below :

$$L_1(x)=[\,1-(x-A_C)/W_r\,] \quad : x\in[\,A_C\,,A_R\,]$$

$$L_2(x)=[\,(x-A_L)/W_l\,] \quad : x\in[\,A_L\,,A_C\,]$$

We can write a fuzzy number A of triangular type with W containing two parts : one is on the left of the center of fuzzy number; the other on the right :

$$A=(A_C\,,W_l\,,W_r)_L$$

And we call a fuzzy number of this type the "fuzzy width-number".

Particularly, when $W_r=W_l$, we have a symmetric fuzzy width-number, and denote the symmetric fuzzy width-number A as follows :

$$A=(A_C\,,W)_L$$

The fuzzy number of this type, in many cases, becomes very convenient and useful for us to carry out operations with fuzzy practical data. In this paper, the fuzzy number of this type is applied to construct the fuzzy hierarchical model.

2.3 Fuzzy Incentive Control

In the resource allocation problem described in this paper, upper-level decision-maker conducts the problem solving on the basis of the information provided from lower-level decision-makers. However lower-level decision-makers tend to exaggerate their profit or capacity of production in order to obtain a larger capital amount. The upper-level decision-maker has to provide the accurate and complete information to the lower-level decision-makers. Fuzzy incentive strategy is utilized to assist in providing the accurate and complete information to the lower-level decision-makers. A number of incentive strategies have been suggested [5,6,7,8,9]. But these incentive strategies still have some room for improvement. The following several problems subsist in the traditional incentive strategy. ①The strategy tends to suppress the more advanced departments which are operating very activity and encourage the less advanced departments. ②A compromise can not be found easily between the upper- and the lower-level decision-makers. ③ The upper-level decision-maker can not derive a satisfying solution while taking into account the volition of the lower-level decision-makers. ④ With the strategy, subordinate decision-makers may receive rewards from the upper-level decision-maker before they begin to work, which does not really obey the incentive principle in management [9]. In this paper, the fuzzy incentive control [10] is proposed to solve the problems described above.

The upper-level decision-maker DM_0 needs to have the accurate and complete information provided by the lower-level decision-maker DM_i . The fuzzy incentive control is utilized to ensure that DM_i provides this accurate information. This control $H_i\left(F_{G_i}\,,F_i\left(\hat{b}_i\right)\right)$ can be described as follows.

$$H_i\left(F_{G_i}\,,F_i\left(\hat{b}_i\right)\right)=F_i\left(\hat{b}_i\right)+e^{ik}\left(F_{G_i}\,,F_i\left(\hat{b}_i\right)\right)$$

In this formula, F_{G_i} and $F_i\left(\hat{b}_i\right)$ are fuzzy numbers. Also, $e^{ik}\left(F_{G_i}\,,F_i\left(\hat{b}_i\right)\right)$ represents the incentive given from

DM_0 to DM_i and can be defined as follows.

$$e^{ik}\left(F_{G_i}, F_i(\hat{b}_i)\right) = e_{\varepsilon}^{ik}\left(F_{G_i}, F_i(\hat{b}_i)\right) * e_p^{ik}\left(F_{G_i}, F_i(\hat{b}_i)\right)$$

In the case, where $e^{ik}\left(F_{G_i}, F_i(\hat{b}_i)\right) = \mu_i\left(\left(F_{G_i} - F_i(\hat{b}_i)\right), h_{\alpha_i}\right)$, and the element $e^{ik}\left(F_{G_i}, F_i(\hat{b}_i)\right)$ is determined by the power relationship between DM_0 and DM_i. The $x*y$ represents the composition of x and y. h_{α_i} shows α -cut.

3. The Hierarchical System and Model

This section will briefly conceptualize a specific problem encountered in the allocation of resources under the hierarchical system as illustrated in Fig.3 [10].

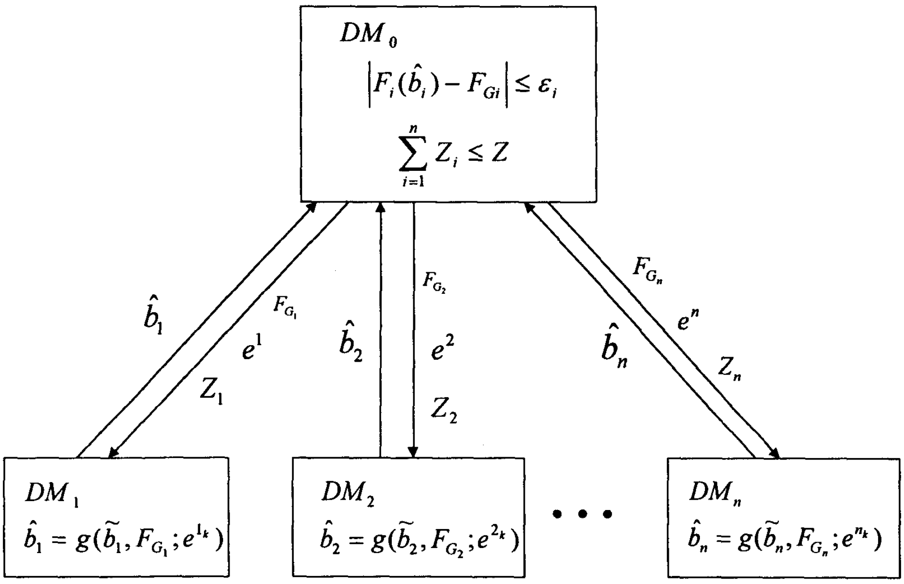

Fig.3 Optimization Problem of Hierarchical System (Application to Resource Allocation Problem)

where Z: total capital amount, Z_i: capital amount assigned to department DM_i, $\hat{b}_i$: minimum capital amount requested from DM_i to DM_o , $\tilde{b}_i$: current capital amount, F_i : maximum profit function of department DM_i estimated by DM_o, $F_i(\hat{b}_i)$: profit predicted by DM_o, F_{Gi} : profit goal given from DM_o to DM_i, e^i : incentive strategy to department DM_i , ε_i : convergence criterion of algorithm.

The ultimate decision in resource allocation is the responsibility of the upper-level manager (as represented by DM_0.). However the decision-maker (DM_i) in level i ($i=1,2,...,n$) not only has the authority to utilize the resources allocated it but is also responsible in providing DM_0 with pertinent information so that DM_0 is able to make the most appropriate decision. This situation could be defined as "DM_0 determines the amount of resources allocated to Department DM_i at the same recognizing the amount of resources allocated the other departments to optimize overall profit." Under this model, DM_0 would allocate the resources as follows. The upper-level decision-maker DM_0 provides the lower-level department with a profit goal. The lower-level decision-maker DM_i advises DM_0 of the amount of resources necessary to achieve the profit goal. The upper-level decision-maker then calculates the amount of profit obtainable based on the amount of resources being requested. If the calculations based on the information provided by the lower-level department indicates a large difference compared with DM_0 's anticipated amount of profit, DM_o provides DM_i with an incentive and DM_i then resubmits the request for resources after revising the amount.

Based on the model described above an optimized hierarchical system algorithm will be formulated.

The Optimized Hierarchical System Algorithm

Step 1 The power relationship between the upper-level decision-maker DM_0 and the lower-level decision-maker $DM_i\,(i=1,2,...,n)$ is set by a paired comparison method.

Step 2 DM_0 provides DM_i with a fuzzy profit goal. $F_{G_i}\ (i=1,2,...,n)$. The repetitive number of k is allotted an initial value of $k = 0$.

Step 3 While taking into account DM_i's current amount of resources $\tilde{b}_i\ (i=1,2,...,n)$, the upper-level decision-maker's profit goal, and the incentive strategy $e^{ik}(i=1,2,...,n)$, the required amount of resources is deduced using the fuzzy inference mechanism. When $k = 0$ the incentive strategy is not applicable.

$$\hat{b}_i = g\left(\tilde{b}_i, F_G\,;e^{ik}\right) \quad (i=1,2,...,n)$$

Step 4 DM_i provides DM_0 with information about the required fuzzy resource amount that was calculated in accordance with step 3.

Step 5 The original fuzzy profit function of $F_i\,(i=1,2,...,n)$ that was determined by DM_0 is substituted with the requested resource amount $\hat{b}_i$. The fuzzy inference mechanism is utilized to calculate the amount of profit that comes with the requested resource amount.$F_i\left(\hat{b}_i\right)\ (i=1,2,...,n)$

Step 6 The difference $\left|F_i\left(\hat{b}_i\right) - F_{G_i}\right|$ between the amounts $F_i\left(\hat{b}_i\right)(i=1,2,...,n)$ and F_{G_i} determined in step 5 is calculated.

$$\left|F_i\left(\hat{b}_i\right) - F_{G_i}\right| \leq \varepsilon_i\,,\ \varepsilon_i \text{ represents the convergence criterion.}$$

When $\left|F_i\left(\hat{b}_i\right) - F_{G_i}\right| \leq \varepsilon_i$, the required resource amount $\tilde{b}_i\,(i=1,2,...,n)$ is $Z_i\,(i=1,2,...,n)$, which is DM_i 's optimized resource allocation amount. In the case where $\sum_{i=1}^{n} Z_i > Z$ (where Z represents the gross amount of resources). Z is normalized by $Z_i\ (i=1,2,...,n)$.

In the case where the convergence criterion cannot be satisfied, the fuzzy inference mechanism is utilized and the incentive strategy reflecting DM_0 's power is provided to $DM_i\,(i=1,2,...,n)$. The process then returns to step 3.

A satisfying solution yielded by applying the algorithm will become the solution with consensus between the upper- and the lower-level decision-makers.

The convergence of algorithm is practically decided on the basis of the following:

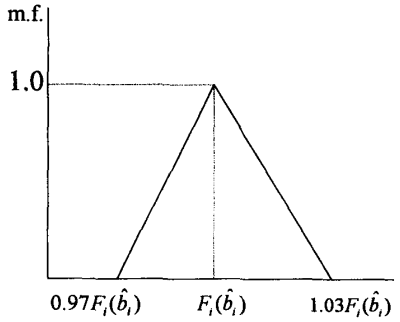

Fig.4.1 Fuzzy Number of Permissible Range of Convergence

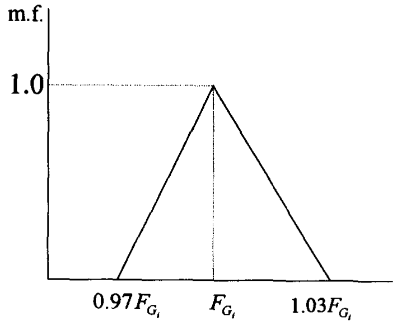

Fig.4.2 Fuzzy Number of Profit Goal

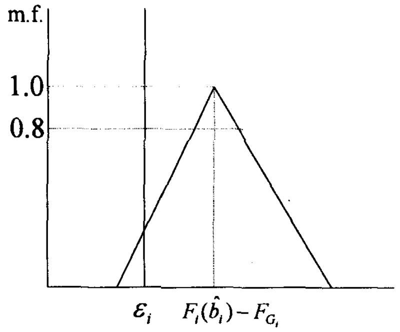

Fig.4.3 $|F_i(\hat{b}_i) - F_{G_i}|$, ε_i **and Convergence Relation(1)**

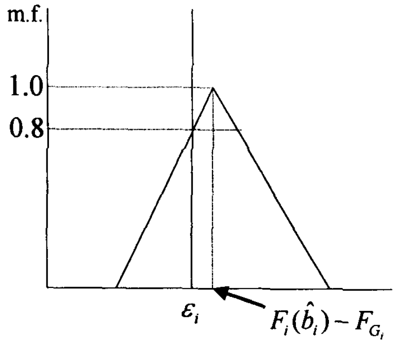

Fig.4.4 $|F_i(\hat{b}_i) - F_{G_i}|$, ε_i **and Convergence Relation(2)**

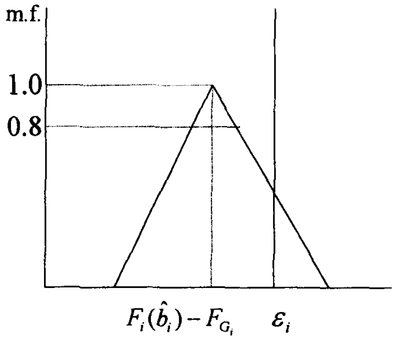

Fig.4.5 $|F_i(\hat{b}_i) - F_{G_i}|$, ε_i **and Convergence Relation(3)**

Fig.4.1 shows an example of the fuzzy number of triangular type with (0.97 $F_i(\hat{b}_i)$, $F_i(\hat{b}_i)$, 1.03 $F_i(\hat{b}_i)$), where closed interval [0.97 $F_i(\hat{b}_i)$, 1.03 $F_i(\hat{b}_i)$] means a permissible range of profit estimated by DM_o. Similarly, Fig.4.2 shows the fuzzy number of triangular type with respect to the profit goal submitted from DM_i to DM_o. The difference between $F_i(\hat{b}_i)$ and F_{G_i} is also the fuzzy number of triangular type. As shown in Figs.4.3 and 4.4 , the algorithm terminates when ε_i is in the interval for α-cut or larger than the value in interval. Further it goes without saying that the algorithm terminates when the center value of difference is smaller than ε_i such as the case shown in Fig.4.5.

Therefore it is clear that the algorithm terminates when either $\left| F_i(\hat{b}_i) - F_{G_i} \right| \leq \varepsilon_i$ or $h_{\alpha_i} \geq \delta$ is satisfied.

In this paper, for example, α -cut(=0.8) is utilized as the convergence criterion. The α -cut value, and the fuzzy numbers with respect to δ and ε_i, in needs, will change depending on the volition of decision-maker concerned with the problem.

Power relationships between upper- and lower-level decision-makers is reflected on the incentive as follows: The incentive is determined by making use of the inference mechanism which gives a suggestion on fluctuation of capital amount requested by the upper-level decision-maker. When the suggestion from the upper-level decision-maker to the lower-level decision-maker is "increase the requested capital", they follow the instructions from the upper-level decision-maker. Otherwise the power is reflected in the relationships between the upper- and the lower-level decision-makers. In this paper, for example, let the powers of the upper- and the lower-level be 0.6 and 0.4 respectively. Further if the incentive from the upper-level decision-maker is "decrease 12 percentage of capital amount" , then the incentive from DM_o to DM_i is normalized as follows:

$$(0.6/(0.6+0.4))\times(-12\%) = -7.2\ (\%)$$

In this example, the incentive "decrease 12 percentage" practically means "decrease 7.2 percentage" reflecting volition of the lower-level decision-maker.

4. Application to a Tangible Problem

Problem: A company needs to allocate resources to its three departments. Each of the three departments will utilize the capital allotted it for operations. The upper-level decision-maker DM_0 ensures that the profit it is to receive from each department equates that of DM_0 's anticipated amount while also allocating an amount of capital that satisfies the decision made by the lower-level decision-makers (DM_1 , DM_2 , DM_3). The three departments are represented by (M_1, M_2, M_3). $DM_i\,(i=1,2,3)$ signifies the profit function contained within each individual department. The capital allocated each department is represented by Z_i with Z being gross capital. When $Z=\sum_{i=1}^{3} Z_i$ it can be anticipated that the amount of capital allocated is at the bare minimum as there is little difference between the fuzzy profit goal and the profit estimated by the lower-level decision-maker.

Premise:

Gross capital: 11,500 (Unit : Ten-thousand Yen)

Current capital amounts of lower departments: $\tilde{b}_1=350$, $\tilde{b}_2=650$, $\tilde{b}_3=200$ (Unit : Ten-thousand Yen)

Fuzzy mechanism:

The fuzzy mechanism consists of seven fuzzy inference systems:

(1) Fuzzy inference system (FIS) with respect to incentive from DM_0 to DM_i (FIS1).

(2) FIS with respect to minimum capital amount requested from DM_1 to DM_0 (FIS2).

(3) FIS with respect to minimum capital amount requested from DM_2 to DM_0 (FIS3).

(4) FIS with respect to minimum capital amount requested from DM_3 to DM_0 (FIS4).

(5) FIS with respect to fuzzy profit goal given from DM_0 to DM_1 (FIS5).

(6) FIS with respect to fuzzy profit goal given from DM_0 to DM_2 (FIS6).

(7) FIS with respect to fuzzy profit goal given from DM_0 to DM_3 (FIS7).

***Optimal process*:**

Step 1 The power relationships between the upper-level decision-maker DM_0 and the lower-level decision-makers $DM_i\,(i=1,2,...,n)$ are set by a paired comparison method.

Table 1 Power relationships between DM_0 and DM_i

	DM_0	DM_1	DM_2	DM_3
DM_0	1.0	0.7	0.5	0.4
DM_1	0.3	1.0	0.5	0.5
DM_2	0.5	0.5	1.0	0.5
DM_3	0.6	0.5	0.5	1.0

Each power of DM_i $(i=0,1,2,3)$ is calculated by making use of eigen vector of Table1.

Power: $DM_0=0.520$, $DM_1=0.449$, $DM_2=0.501$, $DM_3=0.527$

Step 2 DM_0 provides DM_i with a fuzzy profit goal F_{G_i} $(i=1,2,...,n)$. The repetitive number of k is allotted an initial value of $k = 0$.

fuzzy profit goal : $F_{G1}=1{,}000$, $F_{G2}=900$, $F_{G3}=673$ (Unit : Ten-thousand Yen)

Step 3 While taking into account DM_i's current amount of resources b_i $(i=1,2,...,n)$, the upper-level decision-maker's profit goal, and the incentive strategy $e^{ik}(i=1,2,...,n)$, the required amount of resources is deduced using the fuzzy inference mechanism (FIS1, FIS2, FIS3, FIS4). When $k = 0$ the incentive strategy is not applicable.

$\hat{b}_1=g(350,\ 1000\ ;\ e^{10})=3{,}630$, $\hat{b}_2=g(650,\ 900\ ;\ e^{20})=3{,}090$, $\hat{b}_3=g(200,\ 673\ ;\ e^{30})=5{,}060$

(Unit : Ten-thousand Yen)

Step 4 DM_i provides DM_0 with (information about) the required fuzzy resource amount that was calculated in accordance with step 3.

Step 5 The fuzzy inference mechanism (FIS5, FIS6, FIS7) is utilized to calculate the amount of profit that comes with the requested resource amount.$F_i(\hat{b}_i)\,(i=1,2,...,n)$

$F_1(\hat{b}_1)=999$, $F_2(\hat{b}_2)=693$, $F_3(\hat{b}_3)=732$ (Unit : Ten-thousand Yen)

Step 6 Check the convergence criterion, that is, whether or not either $\left|F_i(\hat{b}_i)-F_{G_i}\right| \leq \varepsilon_i$ or $h_{\alpha_i} \geq \delta$ is satisfied.

In the lower department DM_1, the fuzzy profit is shown in Fig.5.1. The fuzzy profit goal is represented by Fig.5.2. Therefore the profit difference is smaller than ε_i(=10) in the lower department DM_1 as shown in Fig.5.3. Then DM_1 's optimized resource allocation amount was obtained. The optimal process on DM_1 terminates. The optimal process on the other lower departments continues. The fuzzy profit, the fuzzy profit goal and the fuzzy number of triangular type of the lower department DM_2 are shown in Fig.5.4, 5.5, 5.6 respectively. The fuzzy profit, the fuzzy profit goal and the profit difference on DM_3 are also formulated as well as DM_1 and DM_2. They will be neglected here.

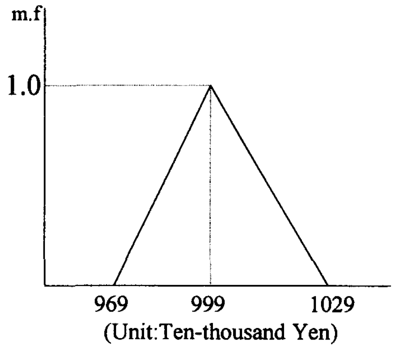

Fig.5.1 Fuzzy Profit of DM_1

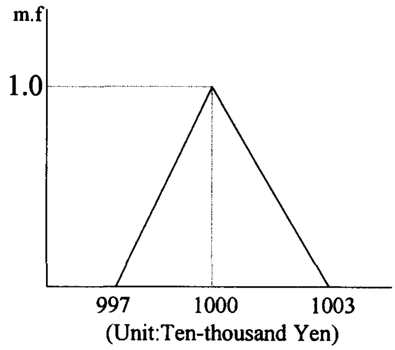

Fig.5.2 Fuzzy Profit Goal of DM_1

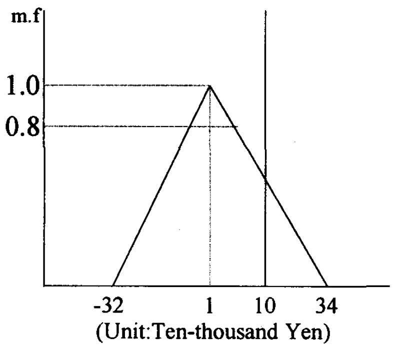

Fig.5.3 Profit Difference of DM_1

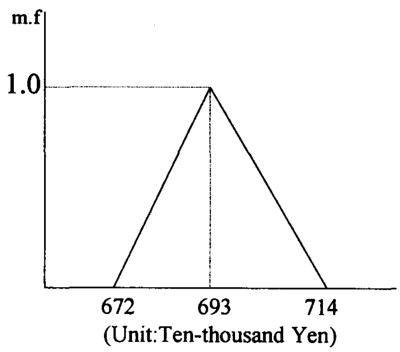

Fig.5.4 Fuzzy Profit of DM_2

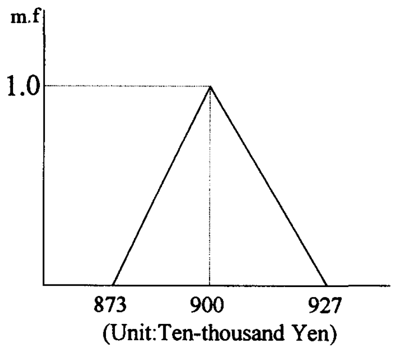

Fig.5.5 Fuzzy Profit Goal of DM_2

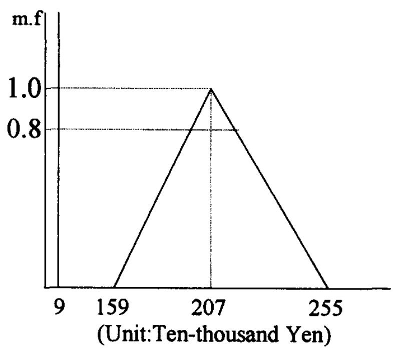

Fig.5.6 Profit Difference of DM_2

α -cut becomes 0.00 because of $\varepsilon_2 = 9$ in Fig.5.6. Therefore the incentive is given from DM_0 to DM_2 and the ratio is as follows.

$$(3{,}090-3{,}500)/3{,}500=-11.7(\%)$$

In a similar way, the incentive is given from the upper decision-maker to DM_3 because of α -cut is 0.00. Then the ratio of incentive is calculated as follows: $(5{,}060-4{,}500)/4{,}500=12.4(\%)$. The fuzzy incentives for the lower departments are practically given by making use of the fuzzy inference mechanism (FIS1) as follows:

$$e^{21}=+7.72\ \%,\ e^{31}=-7.72\ (\%)$$

Further by taking into consideration the power relationships between the upper- and the lower decision-makers, the fuzzy incentive to the lower department is calculated again as follows:

$$e^{31}=(0.5195/(0.5195+0.5270))(-7.72\%)=-3.83\ (\%)$$

e^{31} is given as the real incentive to DM_3 . Let k be k=k+1, the process returns to step 3.

In a similar way, the process is successively carried out. Then the computation results are represented as follows:

Computation results :

DM_1 : $k=0$, $\hat{b}_1=g\ (350,\ 1000\ ;e^{31})=3{,}630$, $F_1(\hat{b}_1)=999$, $h_{\alpha 1}\leq \varepsilon_1\ (=10)$: convergence

DM_2 : $k=2$, $\hat{b}_2=g\ (650,\ 900\ ;\ 5.33\%)=3{,}506$, $F_2(\hat{b}_2)=902$, $\varepsilon \leq \varepsilon_2\ (=9)$: convergence

DM_3 : $k=5$, $\hat{b}_3=g\ (200,\ 673\ ;\ -0.50\%)=4{,}602$, $F_3(\hat{b}_3)=685$, $h_{\alpha 3}(=0.88)\geq h_\alpha(=0.80)$: convergence

From this, each of the required capital amount of lower-level departments is expressed as follows:

$\hat{b}_1=3{,}630$, $\hat{b}_2=3{,}506$, $\hat{b}_3=4{,}602$, (Unit : Ten-thousand Yen)

Overall capital amount required : $\sum_{i=1}^{3}\hat{b}_i=11{,}738$

Normalized optimal capital amount $z_i^*(i=1,2,3)$ assigned to DM_i :

$z_1^*=3{,}556$, $z_2^*=3{,}435$, $z_3^*=4{,}506$ (Unit : Ten-thousand Yen)

The results show a reasonable solution (satisfying solution) directly reflecting knowledge, emotion, experience and volition of upper- and lower-level decision-makers. It can be said that the method presented here grasps the real meaning of the optimization for ill-defined problems. The method reasonably induces the solution which upper- and lower-level decision-makers estimated.

5. Conclusion

Attempting to translate mathematically the incentive elements of human ego, experience, and volition, in the process of decision making, is prone to be difficult. As such there has been a tendency to avoid focusing on this element in the equation. This paper formalized the hierarchical system problem through the development of a model.

Further, an algorithm, which attempts to fill the void by formulating the model, was proposed as an approach to solving ill-defined problems. The model and the algorithm were applied to a tangible problem that involved the allocation of resources by business management. The algorithm recognized and effectively integrated the elements void in the traditional optimization -i.e. volition, knowledge and experience of decision-maker. Because of this, it can be said that this new method is more effective than the traditional models in a practical sense. The characteristics of the proposed method is summarized as follows: ①We can effectively get a preferred solution (satisfying solution) reflecting volition, knowledge and experience of decision-maker without doing the traditional optimization. ②The incentive is provided while taking into account the lower- and the upper-level decision-maker's situations, and the preferred solution with consensus is reasonably obtained by the model and the algorithm presented here. The model and the algorithm are also applicable to solve other two-level system problems with non-production institutions.

On the other hand, the model still has some room for improvement with respect to the fuzzy numbers of the fuzzy profit goal, and the fuzzy profit. Further, in order to inspect the effectiveness of the proposed model, we will have to perform a large number of simulations on more practical problems.

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