The Basis for a Game–theoretic Analysis of Wholesale Electricity Markets

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Abstract

Starting from a simple oligopoly model for electricity markets, we prove the theorems that are the base for Tezuka and Ishii (2006), and Tezuka and Ishii (2007). By applying the theorems, it is relatively easy to obtain the Nash equilibrium in some examples. Additionally we show some extensions of the model.

1 Introduction

In recent years, a number of wholesale power markets have been deregulated and the amount of transactions conducted is rapidly increasing. Therefore, market participants and researchers have analysed various interesting characteristics of electricity price behavior. Their purposes are risk management, designing market mechanisms, and economic policies.

Johnson and Barz (1999) estimate parameters of eight stochastic processes with electricity price data, and compare them by simulations. As those models are not so complicated, it is relatively easy to use them to measure the price risk. But they do not consider the demand–supply mechanism.

Barlow (2002), and Kanamura and Ohashi (2006) present spot price models based on the relationship between demand and supply. Barlow (2002) applies inverse functions of the Box–Cox transformations to the marketwide supply function. Kanamura and Ohashi (2006) assume a family of hockey–stick shaped functions as the merit order curve. Both reports estimate the parameters from market data. Those models capture well some features of electricity prices. However, neither of them explain the construction of the market wide supply function.

Bessembinder and Lemmon (2002) construct the marketwide supply curve by the sum of individual marginal cost curves, and derive electricity spot and forward price formulae in a one–period trading model with an equilibrium approach. In the model it is assumed that power producers and retailing firms are homogeneous. Ishii and Tezuka (2005) generalize the Bessembinder and Lemmon (2002) model from the following view points: non–homogeneity of market participants and multi–period trading times. However these models ignore the strategic behavior of power generating firms.

Klemperer and Meyer (1989) is the most prominent article regarding price determination where each firm can strategically offer its supply curve. It is called the supply function approach. Although the electricity demand uncertainty is assumed in the supply function approach, it is only used to derive market supply function. Therefore, strategies and market prices would not be affected by the demand probability distribution. Green and Newbery (1992) and Newbery (1998) employ the supply function approach to analyse electricity spot prices. In contrast to the supply function approach, Tezuka and Ishii (2006), and Tezuka and Ishii (2007) construct another kind of supply function approach. It is a non–cooperative game theoretic pricing framework. In the model, the electricity demand affects profit through the individual supply curves, and each firm decides its supply curve to maximize the α–quantile of its profit distribution along with taking other generators' strategies into consideration.

In connection with electricity forward, we also notice that Eydeland and Geman (1999) propose a different valuation framework for power forward prices from the equilibrium approach.

The structure of this paper is as follows: Section 2 presents the model. In section 3, we prove the theorems that play an important role in Tezuka and Ishii (2006), and Tezuka and Ishii (2007). We show some extensions of the model in section 4. We conclude our results and mention future research in section 5.

2 The Model

In this section, we describe the assumptions and notations used throughout this paper. These are common with those used in Tezuka and Ishii (2006), and Tezuka and Ishii (2007).

There are power generating firms and retailers in a wholesale spot market. Power generating firms supply electricity to retailers who are buyers in the market and who distribute power to their customers. We assume that there are no speculators.

For simplicity, our model is a one–period model; however, it is easily extended to a multi–period model because of the non–storability and balancing rules in electricity spot markets. We shall describe the details in section 4.

Let $(\Omega, \mathcal{F}, P)$ be a probability space. A non–negative valued random variable Y is called the total electricity demand at time 1. $M \in \mathrm{N}$ is interpreted as the number of power generating firms. For each $j = 1, 2, \dots, M$, $f_j : [0, +\infty) \to [0, +\infty)$ is supposed to be continuous and strictly increasing, and $\lim_{x\to\infty} f_j(x) = +\infty$. We define $C_j : [0, +\infty) \to [0, +\infty)$ as

$$C_j\,(x) = \eta_j + \int_0^x f_j\,(u)du \;\text{ for } x \in [0, +\infty), \tag{1}$$

where η_j is a positive constant. We use C_j as the cost function of power generating firm j and f_j as the marginal cost function.

For all $j = 1, 2, ..., M$, given a subset $A_j \subset R^n$ that contains 0, let $g_j : [0, +\infty) \times A_j \to [0, +\infty)$ be a continuous function with the following three conditions:

$g_j\,(x, 0) = f_j\,(x)$ for $x \in [0, +\infty)$,
For each $\lambda_j \in A_j$, $g_j\,(x, \lambda_j)$ is strictly increasing with respect to x,
$g_j\,(x, \lambda_j) \geq f_j\,(x)$ for $(x, \lambda_j) \in [0, +\infty) \times A_j$.

We give the economic interpretation on these functions. A_j represents all strategies of power producer j. With the distribution of Y and information of the other power producers' strategies, power producer j decides a strategy λ_j, and bids its supply functions $g_j\,(x, \lambda_j)$ to the market at time 0.

In addition, for each $\lambda_j \in A_j$, we define $g_j^{-1}\,(z, \lambda_j)$ as follows:

$$g_j^{-1}(z, \lambda_j) = \begin{cases} 0 \text{ for } z \leq g_j\,(0, \lambda_j) \\ x \text{ satisfying } z = g_j\,(x, \lambda_j) \text{ for } z > g_j\,(0, \lambda_j), \end{cases} \tag{2}$$

where $g_j^{-1}\,(z, 0)$ is also denoted by $f_j^{-1}\,(z)$. From the above assumptions, it is easily to show that $\lim_{x\to\infty} g_j\,(x, \lambda_j) = +\infty$ for each $\lambda_j \in A_j$. Then, the rane of $g_j\,(\cdot, \lambda_j)$ is $[g_j\,(0, \lambda_j), \infty)$ and the domain of $g_j^{-1}\,(\cdot, \lambda_j)$ is $[0, \infty)$.

Additionally, we define

$$G(z, \lambda) := \sum_{j=1}^{M} g_j^{-1}(z, \lambda_j),$$

where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_M)$. $G(z, \lambda)$ is interpreted as the merit order curve or the marketwide supply curve in the case that the strategies of M power producers are $\lambda_1, \lambda_2, ..., \lambda_M$.

3 Fundamental Properties

Starting from the assumptions stated above, we show the fundamental theorems.

Lemma 1 For each $j = 1, 2, \dots, M$, g_j^{-1} is continuous, and $\lim_{z\to\infty} g_j^{-1}\,(z, \lambda_j) = \infty$.

PROOF. First, we prove the continuity for the case in which $\lambda_j \in A_j$ and $a > g_j\,(0, \lambda_j)$.

Let $b := g_j^{-1}\,(a, \lambda_j)$. Clearly $b > 0$. For any $\varepsilon \in (0, b)$, by the assumption that $g_j\,(x, \lambda_j)$ is strictly increasing with respect to x,

$$g_j\,(b - \varepsilon, \lambda_j) < g_j\,(b, \lambda_j) < g_j\,(b + \varepsilon, \lambda_j).$$

Then, we can choose a' and a'' such that

$$g_j\,(b-\varepsilon,\,\lambda_j) < a' < g_j\,(b,\,\lambda_j) < a'' < g_j\,(b+\varepsilon,\,\lambda_j).$$

Since g_j is continuous, there is a $h(\varepsilon) > 0$ such that $g_j\,(b-\varepsilon,\,u) < a'$ and $a'' < g_j\,(b+\varepsilon,\,u)$ for all $u \in A_j$ with $|\,u-\lambda_j\,| < h(\varepsilon)$. Hold any $u \in \{v \in A_j\,|\,|\,v-\lambda_j\,| < h(\varepsilon)\}$ fixed, and choose $\delta > 0$ such that

$$a' < a-\delta < a < a+\delta < a'',$$

then, for any $z \in (\mathrm{a}-\delta,\,\mathrm{a}+\delta)$,

$$g_j\,(b-\varepsilon,\,u) < z < g_j\,(b+\varepsilon,\,u).$$

By the assumption that $g_j\,(x,\,u)$ is strictly increasing and continuous with respect to x, there exists a $x_{z,u} \in (b-\varepsilon,\,b+\varepsilon)$ such that $z = g_j\,(x_{z,u},\,u)$. Thus,

$$\left|g_j^{-1}(z,u)-b\right| < \varepsilon.$$

This proves that g_j^{-1} is continuous at $(a,\,\lambda_j)$.

Now suppose $a \leq g_j\,(0,\,\lambda_j)$. Then, $g_j^{-1}\,(a,\,\lambda_j) = 0$. For $\forall\varepsilon > 0$,

$$g_j\,(0,\,\lambda_j) < g_j\,(\varepsilon,\,\lambda_j).$$

Then, by choosing $a' \in (g_j\,(0,\,\lambda_j),\,g_j\,(\varepsilon,\,\lambda_j))$, there is a $h(\varepsilon) > 0$ such that $a' < g_j\,(\varepsilon,\,u)$ for all $u \in A_j$ with $|\,u-\lambda_j\,| < h\,(\varepsilon)$. If we choose $\delta \in (0,\,a'-a)$, then,

$$0 < z < g_j\,(\varepsilon,\,u)$$

whenever $z \in [0,\,a+\delta)$. By the definition of g_j^{-1}, if $z \leq g_j\,(0,\,u)$, we have $0 = g_j^{-1}\,(z,\,u)$, otherwise, there is a number $x_z \in (0,\,\varepsilon)$ such that $z = g_j\,(x_z,\,u)$. Therefore,

$$0 \leq g_j^{-1}\,(z,\,u) < \varepsilon,$$

i.e., g_j^{-1} is continuous at $(a,\,\lambda_j)$.

The latter assertion remains to be shown. From the assumption that $g_j\,(x,\,\lambda_j)$ is strictly increasing in x, for $\forall c \geq 0$ and $\forall x > c$,

$$g_j\,(x,\,\lambda_j) > g_j\,(c,\,\lambda_j).$$

Then,

$$g_j^{-1}\,(z,\,\lambda_j) > c,$$

whenever $z > g_j\,(c,\,\lambda_j)$. So, $g_j^{-1}\,(z,\,\lambda_j) \to \infty$ as $z \to \infty$.

Lemma 2 The function G has the following properties:

(i) G is continuous;

(ii) Given any $\lambda \in \prod_{i=1}^{M} A_i$, $G(z, \lambda)$ is increasing with respect to z;

(iii) $G(z, \lambda) = 0$ on $\left\{ (z,\lambda) \middle| z \geq 0, \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_M) \in \prod_{i=1}^{M} A_i, z \leq \min_{i=1,2,\ldots,M} (g_i(0, \lambda_i)) \right\}$;

(iv) Given any $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_M) \in \prod_{i=1}^{M} A_i$, $G(z, \lambda)$ is strictly increasing on

$\{z \mid z \geq \min_{i=1,2,\ldots,M} (g_i(0, \lambda_i))\}$;

(v) $\lim_{z \to \infty} G(z, \lambda) = \infty$ for each $\lambda \in \prod_{i=1}^{M} A_i$.

Proof. It is easy to see that (i), (ii), (iii) and (v) hold from Lemma 1 and the definition of G. So only (iv) is left to us.

Suppose that $z' > z \geq \min_{i=1,2,\ldots,M} (g_i(0, \lambda_i))$. Then there is a $k = 1, 2, \ldots, M$ such that

$$g_k^{-1}(z, \lambda_k) < g_k^{-1}(z', \lambda_k),$$

and for $\forall j = 1, 2, \ldots, M$

$$g_j^{-1}(z, \lambda_j) \leq g_j^{-1}(z', \lambda_j),$$

therefore,

$$G(z) < G(z').$$

Lemma 3 Given $\forall y > 0$ and $\forall \lambda \in \prod_{i=1}^{M} A_i$, there exists a unique solution of the equation $G(z, \lambda) = y$ in z, and the solution is greater than $\min\{g_1(0, \lambda_1), g_2(0, \lambda_2), \ldots, g_M(0, \lambda_M)\}$.

Proof. This is an immediate consequence of Lemma 2.

Hence, the solution will be denoted by $\varphi(y, \lambda)$ hereafter. For each $j = 1, 2, \ldots, M$, without knowledge of the realization Y, M power producers bid their supply functions $g1(x, \lambda_1)$, $g_2(x, \lambda_2)$, ..., $g_M(x, \lambda_M)$, respectively, to the electricity spot market at time 0. Then, the market maker aggregates the supply functions to construct the merit order curve $G(z, \lambda)$. At time 1, all market participants observe a realized electricity demand y, and the electricity spot price must be determined such that total supply is equal to y. Therefore, $\varphi(y, \lambda)$ is labelled as the electricity spot price, and $g_j^{-1}(\varphi(y, \lambda), \lambda_j)$ is interpreted as the supply of power producer j. Moreover, we define $F : (0, \infty) \times \prod_{i=1}^{M} A_i \to \mathrm{R}$ as follows:

$$F_j(y,\lambda)=\varphi(y,\lambda)\cdot g_j^{-1}(\varphi(y,\lambda),\lambda_j)-C_j(g_j^{-1}(\varphi(y,\lambda),\lambda_j)). \tag{3}$$

Here, F_j represents the profit of generator j.

Lemma 4 $\varphi:(0,\infty)\times\prod_{i=1}^{M}A_i\to(0,\infty)$ is continuous.

Proof. Fix $\forall y>0$ and $\forall\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_M)\in\prod_{i=1}^{M}A_i$. It is suffcient to show that the sequence $\{\varphi(y_n,l_n)\}_{n\in\mathrm{N}}$ converges to $\varphi(y,\lambda)$ whenever the sequence $\{(y_n,l_n)\}_{n\in\mathrm{N}}$ converges to (y,λ), where $l_n=(\lambda_{n,1},\lambda_{n,2},\ldots,\lambda_{n,M})$. Let $z'=\limsup_{n\to\infty}\varphi(y_n,l_n)$ and $z''=\liminf_{n\to\infty}\varphi(y_n,l_n)$. Here, we note that both $z', z''\geq\min\{g_1(0,\lambda_1), g_2(0,\lambda_2),\ldots,g_M(0,\lambda_M)\}$. We will show that $z'=z''$. Clearly $z'\geq z''$. We shall assume $z'>z''$, and deduce a contradiction.

In fact, $\{\varphi(y_n,l_n)\}_{n\in\mathrm{N}}$ contains a subsequence $\{\varphi(y_{n_k},l_{n_k})\}_{k\in\mathrm{N}}$ such that

$$z'=\lim_{k\to\infty}\varphi(y_{n_k},l_{n_k}),$$

and a subsequence $\{\varphi(y_{m_k},l_{m_k})\}_{k\in\mathrm{N}}$ such that

$$z''=\lim_{k\to\infty}\varphi(y_{m_k},l_{m_k}),$$

For each $k\in\mathrm{N}$, we set $z_{n_k}=\varphi(y_{n_k},l_{n_k})$ and $z_{m_k}=\varphi(y_{m_k},l_{m_k})$. Then,

$$G(z_{n_k},l_{n_k})=y_{n_k}.$$

Since G is continuous, we can take the limit as $k\to\infty$, obtaining

$$G(z',\lambda)=y.$$

Similarly, we have

$$G(z'',\lambda)=y.$$

Therefore, we get

$$G(z',\lambda)=y=G(z'',\lambda).$$

But this contradicts Lemma 2.

Lemma 5 Given any $\lambda\in\prod_{i=1}^{M}A_i$, $\varphi(y,\lambda)$ is strictly increasing with respect to y.

Proof. Let $y'<y''$ be two numbers in $(0,\infty)$. By Lemma 3,

$$G(\varphi(y',\lambda),\lambda)=y'<y''=G(\varphi(y'',\lambda),\lambda),$$

and

$$\varphi(y', \lambda), \varphi(y'', \lambda) > \min\{g_1(0, \lambda_1), g_2(0, \lambda_2), \dots, g_M(0, \lambda_M)\}.$$

Then, we have

$$\varphi(y', \lambda) < \varphi(y'', \lambda).$$

Lemma 6 For each $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_M) \in \prod_{i=1}^{M} A_i$, $\lim_{y\to\infty} \varphi(y, \lambda) = \infty$.

Proof. By Lemma 2 (iv), for $\forall c > \min\{g_1(0, \lambda_1), g_2(0, \lambda_2), \dots, g_M(0, \lambda_M)\}$ and $\forall z > c$,

$$G(z, \lambda) > G(c, \lambda).$$

Then $\varphi(y, \lambda) > c$ for any $y > G(c, \lambda)$, and the desired conclusion follows.

Theorem 1 Given any $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_M) \in \prod_{i=1}^{M} A_i$, $F_j(y, \lambda)$ is continuous and increasing with respect to y. In particular, $F_j(y, \lambda)$ is strictly increasing on $\{y > 0 \mid \varphi(y, \lambda) \geq g_j(0, \lambda_j)\}$.

Proof. The continuity is a consequence of Lemma 1 and Lemma 4.

We now prove that $F_j(y, \lambda)$ is a increasing function in y. Let $y < y'$ be two numbers in $(0,\infty)$. By Lemma 5,

$$\varphi(y, \lambda) < \varphi(y', \lambda).$$

For the case in which $\varphi(y', \lambda) \leq g_j(0, \lambda_j)$,

$$g_j^{-1}(\varphi(y, \lambda), \lambda_j) = g_j^{-1}(\varphi(y', \lambda), \lambda_j) = 0,$$

therefore we have

$$F_j(y, \lambda) = F_j(y', \lambda) = -\eta_j.$$

We will consider the case $\varphi(y', \lambda) > g_j(0, \lambda_j)$. From Lemma 1 and Lemma 5, it follows that

$$g_j^{-1}(\varphi(y, \lambda), \lambda_j) < g_j^{-1}(\varphi(y', \lambda), \lambda_j).$$

Here, it is clear that $\varphi(y', \lambda) = g_j(g_j^{-1}(\varphi(y', \lambda), \lambda_j), \lambda_j)$. Hence

$$
\begin{aligned}
&\varphi(y',\lambda)\left\{g_j^{-1}(\varphi(y',\lambda),\lambda_j)-g_j^{-1}(\varphi(y,\lambda),\lambda_j)\right\}\\
&=\int_{g_j^{-1}(\varphi(y,\lambda),\lambda_j)}^{g_j^{-1}(\varphi(y',\lambda),\lambda_j)}\varphi(y',\lambda)du\\
&>\int_{g_j^{-1}(\varphi(y,\lambda),\lambda_j)}^{g_j^{-1}(\varphi(y',\lambda),\lambda_j)}g_j(u,\lambda_j)du\\
&\geq\int_{g_j^{-1}(\varphi(y,\lambda),\lambda_j)}^{g_j^{-1}(\varphi(y',\lambda),\lambda_j)}f_j(u)du.
\end{aligned}
\tag{4}
$$

By the definition of C_j,

$$
C_j(g_j^{-1}(\varphi(y',\lambda),\lambda_j))-C_j(g_j^{-1}(\varphi(y,\lambda),\lambda_j))=\int_{g_j^{-1}(\varphi(y,\lambda),\lambda_j)}^{g_j^{-1}(\varphi(y',\lambda),\lambda_j)}f_j(u)du. \tag{5}
$$

Therefore,

$$
\begin{aligned}
&F_j(y',\lambda)-F_j(y,\lambda)\\
&=\{\varphi(y',\lambda)-\varphi(y,\lambda)\}\,g_j^{-1}(\varphi(y,\lambda),\lambda_j)\\
&\qquad+\varphi(y',\lambda)\{g_j^{-1}(\varphi(y',\lambda),\lambda_j)-g_j^{-1}(\varphi(y,\lambda),\lambda_j)\}\\
&\qquad-\{C_j(g_j^{-1}(\varphi(y',\lambda),\lambda_j))-C_j(g_j^{-1}(\varphi(y,\lambda),\lambda_j))\}\\
&>\{\varphi(y',\lambda)-\varphi(y,\lambda)\}\,g_j^{-1}(\varphi(y,\lambda),\lambda_j)\geq 0,
\end{aligned}
$$

where we have used (4) and (5). We use this fact to draw the desired conclusions.

Theorem 2 For fixed $\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_M)\in\prod_{i=1}^{M}A_i$, $\lim_{y\to\infty}F_j(y,\lambda)=\infty$.

Proof. For any $y\in\{y'>0\mid\varphi(y',\lambda)>g_j(0,\lambda_j)\}$, we have

$$
\varphi(y',\lambda)=g_j(g_j^{-1}(\varphi(y',\lambda),\lambda_j),\lambda_j), \tag{6}
$$

and

$$
g_j(g_j^{-1}(\varphi(y',\lambda),\lambda_j),\lambda_j)\geq f_j(g_j^{-1}(\varphi(y',\lambda),\lambda_j))\geq f_j(x) \tag{7}
$$

where $x\in[0,g_j^{-1}(\varphi(y',\lambda),\lambda_j)]$. Applying (6) and (7) yields

$$F_j(y,\lambda) = \int_0^{g_j^{-1}(\varphi(y,\lambda),\lambda_j)} \varphi(y,\lambda) - f_j(x)dx - \eta_j$$

$$= \int_0^{g_j^{-1}(\varphi(y,\lambda),\lambda_j)} g_j(g_j^{-1}(\varphi(y',\lambda),\lambda_j),\lambda_j) - f_j(x)dx - \eta_j$$

$$\geq \int_0^{g_j^{-1}(\varphi(y,\lambda),\lambda_j)} f_j(g_j^{-1}(\varphi(y',\lambda),\lambda_j)) - f_j(x)dx - \eta_j$$

$$= \int_0^{f_j(g_j^{-1}(\varphi(y,\lambda),\lambda_j))} f_j^{-1}(z)dz - \eta_j.$$

By using Lemma 1 and Lemma 6, we can conclude that $F_j(y, \lambda) \to \infty$ as $y \to \infty$.

Here, for each $j = 1, 2, \ldots, M$, we define $q_j : \prod_{i=1}^{M} A_i \times (0, 1) \to \mathrm{R}$ as follows:

$$q_j(\lambda, \alpha) = \inf \{u \in \mathrm{R} \mid P(F_j(Y, \lambda) \leq u) \geq \alpha\}. \tag{8}$$

That is to say, $q_j(\lambda, \alpha)$ represents the α–quantile (quantile of order α) of power producer j's profit in the case that the strategies of M generators are $\lambda_1, \lambda_2, \ldots, \lambda_M$.

Theorem 3 Let Y be a continuous random variable, and y_α denote the α–quantile of Y. Then, for $\forall j = 1, 2, \ldots, M$ and $\forall \lambda \in \prod_{i=1}^{M} A_i$,

$$q_j(\lambda, \alpha) = F_j(y_\alpha, \lambda). \tag{9}$$

Proof. The assertion follows from Theorem 1.

Next we define a non–cooperative game as follows: for each $j = 1, 2, \ldots, M$,

$$\begin{cases} \text{the strategy set for player } j \text{ is } A_j, \\ \text{the payoff to player } j \text{ is } q_j(\lambda, \alpha). \end{cases} \tag{10}$$

In Tezuka and Ishii (2006), and Tezuka and Ishii (2007), if there exists a unique Nash equilibrium λ^*, $\varphi(Y, \lambda^*)$ is referred to as the equilibrium spot price at time 1.

In some examples of the game (10) we can find the unique Nash equilibrium. Theorem 3 makes the calculation easy. See Tezuka and Ishii (2006), and Tezuka and Ishii (2007) for details.

4 Extensions of the Model

4.1 A Capacity Model

It is said that a typical generator's supply curve is a slight upward slope (or flat) until the point of maximum generation. Then it jumps to infinity above its maximum output level.

That is, the generator has a capacity constraint. Then we construct a model that expresses this supply curve shape

Let $r_1, r_2, \dots, r_M$ be positive constants. Suppose that $\tilde{f}_j : [0, r_j) \to [0, \infty)$ is continuous and strictly increasing, and $\lim_{x \to r_j} \tilde{f}_j(x) = +\infty$ for each $j = 1, 2, \dots, M$. We use $\tilde{f}_j$ as producer j's marginal cost function instead of f_j. Moreover, let $\tilde{g}_j : [0, r_j) \times A_j \to [0, +\infty)$ be a continuous function with the following three conditions:

$\tilde{g}_j(x, 0) = \tilde{f}_j(x)$ for $x \in [0, r_j)$,

For each $\lambda_j \in A_i$, $\tilde{g}_j(x, \lambda_j)$ is strictly increasing with respect to x,

$\tilde{g}_j(x, \lambda_j) \geq \tilde{f}_j(x)$ for $(x, \lambda_j) \in [0, r_j) \times A_j$.

Here, $\tilde{g}_j$ plays an instead role of g_j.

In this set–up, all properties, which correspond to the lemmas and theorems proved in section 3, also hold with slight modification.

4.2 A Multi–period Model

We describe an extension to a multi–period model. Since electricity is non–storable, in general, most power generating firms can not supply electricity produced today at a later day. We take this restriction into consideration.

Suppose that $\{\mathcal{F}_t \mid t \in Z^+\}$ is a filtration on $(\Omega, \mathcal{F}, P)$, and a stochastic process $\tilde{Y}$ is adapted to $\{\mathcal{F}_t \mid t \in Z^+\}$. For any $t \in Z^+$, $\tilde{Y}(t)$ is interpreted as the electricity demand at time t. Moreover we define

$$\tilde{q}_j(t, \lambda, \alpha) = \inf\{u \in R \mid P(F_j(\tilde{Y}(t), \lambda) \leq u \mid \mathcal{F}_t) \geq \alpha\},$$

for $t \in Z^+$, $\lambda \in \prod_{i=1}^{M} A_i$, and $\alpha \in (0, 1)$. Thus we define the following non–cooperative game: for $\forall t \in Z^+$ and $\forall j = 1, 2, \dots, M$,

$$\begin{cases} \text{the strategy set for player } j \text{ is } A_i, \\ \text{the payoff to player } j \text{ is } \tilde{q}_j(t+1, \lambda, \alpha). \end{cases} \tag{11}$$

We explain the game (11). At time t, producer j can not use the electricity that it has produced at time $s < t$, to manage the time t profit. That is, when producer j decides the strategy for time $t + 1$ profit, it does not consider the output before time $t + 1$. Then, the supply curve, which producer j offers to the marketmaker at time t, only depends on the distribution of $\tilde{Y}(t+1)$ under the condition $\mathcal{F}_t$ and other strategies. For the reasons stated above, we use $F_j(\tilde{Y}(t+1), \lambda)$ as the time $t + 1$ profit of producer j. The game (11) represents the situation that each generator tries to maximize the α–quantile of time t + 1 profit distribution at time t.

In this set–up, the property similar to Theorem 3 holds.

4.3 Other Extensions

There are much more extensions of the model.

Let c be a positive number and $\beta \in (0, 1)$. Then, we consider the following non–cooperative game: for each $j = 1, 2, \dots, M$,

$$\begin{cases} \text{the strategy set for player } j \text{ is } B_j = \{\lambda_j \in A_j \mid q_j(\lambda, \beta) \geq c\}, \\ \text{the payoff to player } j \text{ is } q_j(\lambda, \alpha). \end{cases} \tag{12}$$

We set $\alpha = 0.5$ and $\beta = 0.01$ to explain the game (12) specifically. c is interpreted as the worst case profit that is expected to occur at 1 percent probability. It is similar to VaR. Then, each member of B_j is a strategy such that the 1 percent worst case profit is greater than or equal to c. Generator j will choose the strategy maximizing the median of the profit distribution from B_j.

In the base model, we assume that every generator applies the same level of α to its payoff function. We can loosen this assumption. It is far more natural that the levels of α are different among generators. That is to say, $q_j(\lambda, \alpha)$ in the game (10) is replaced by $q_j(\lambda, \alpha_j)$, where $\alpha_1, \alpha_2, \dots, \alpha_M \in (0, 1)$.

In the multi–period model, we assume that the marginal cost functions are time invariant. Since fuel prices, especially gas and oil prices, change stochastically, it is general that the marginal cost functions fluctuate over time. So we can extend the model to a stochastic marginal cost function model. However we do not describe such details in this paper.

5 Conclusion

In this paper we describe the proofs of the fundamental theorems reported by Tezuka and Ishii (2006), and Tezuka and Ishii (2007). We also show some extensions of the base model. In these models, the theorems still play important roles to derive Nash equilibria. Some problems remain for our future research. First, we have not yet proved the existence of Nash equilibria in a general oligopoly model. As well, it is necessary to analyse electricity spot price data with some specific models.

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